Answers - Sequences

Year: Year 10

Answer to Question 1

a. **21, 25**. Add 4 each time. The difference between consecutive terms is always 4, so this is an arithmetic sequence: $17 + 4 = 21$ and $21 + 4 = 25$.
b. **5, 2.5**. Divide by 2 (halve) each time. Each term is half of the one before, so the next terms are $10 \div 2 = 5$ and $5 \div 2 = 2.5$.
c. **162, 486**. Multiply by 3 each time. Each term is 3 times the one before, so this is a geometric sequence: $54 \times 3 = 162$.
d. **25, 36**. These are the square numbers $1^2, 2^2, 3^2, \ldots$ The differences are 3, 5, 7, so the next differences are 9 and 11: $16 + 9 = 25$ and $25 + 11 = 36$.

Answer to Question 2

a. Common difference 4, so $4n + \ldots$: $4 \times 1 = 4$, $7 - 4 = 3$. $4n + 3$ The common difference tells us the number in front of n. The zero term (the term before the first) is $7 - 4 = 3$. Check: $4 \times 2 + 3 = 11$.
b. Difference -3: $-3n + \ldots$, $20 - (-3) = 23$. $23 - 3n$ The terms go down by 3, so the coefficient of n is -3. The zero term is $20 + 3 = 23$. Check: $23 - 3 \times 2 = 17$.
c. Difference 5: $5n + \ldots$, $-3 - 5 = -8$. $5n - 8$ The terms go up by 5, and the zero term is $-3 - 5 = -8$. Check: $5 \times 3 - 8 = 7$.
d. $4 \times 50 + 3 =$ **203** Substitute $n = 50$ into $4n + 3$. The 50th term is 203.

Answer to Question 3

a. $n = 1, 2, 3, 4$: **3, 8, 13, 18** Substitute $n = 1$, 2, 3 and 4 into $5n - 2$; the terms go up by 5 each time.
b. **4, 7, 12, 19** Square n first, then add 3: $1 + 3$, $4 + 3$, $9 + 3$, $16 + 3$.
c. **2, 4, 8, 16** This is a geometric sequence where each term is double the one before.
d. **2, 6, 12, 20** Multiply each n by the next whole number: 1×2, 2×3, 3×4, 4×5.

Answer to Question 4

a. $3n + 1 = 100$, $3n = 99$, $n = 33$. **Yes, the 33rd term.** Set the nth term equal to the number and solve for n. A number is a term only if n is a positive whole number.
b. $4n - 2 = 150$, $4n = 152$, $n = 38$. **Yes, the 38th term.**
c. $6n + 1 = 75$, $6n = 74$, $n = 12.33$, which is not a whole number. **No.**

d. $4n + 3 = 203$, $4n = 200$, $n = 50$. **Yes, the 50th term.**

Answer to Question 5

a. Ratio **2**. 10th term $= 3 \times 2^9 = 3 \times 512 =$ **1536** Each term is double the one before. The nth term of a geometric sequence is $a \times r^{n-1}$, so the 10th term is 3×2^9.
b. Ratio $\frac{1}{2}$. 6th term $= 160 \times \left(\frac{1}{2}\right)^5 =$ **5** Each term is half the one before, so the ratio is $\frac{1}{2}$. The 6th term is $160 \times \left(\frac{1}{2}\right)^5 = 160 \div 32$.
c. Terms: 81, 243, 729, **2187**. The first term bigger than 1000 is **2187**. Keep multiplying by 3: 729 is still less than 1000, so the first term bigger than 1000 is 2187.
d. Ratio -2. Next terms: **−16, 32**. Each term is multiplied by -2, so the signs alternate between positive and negative.

Answer to Question 6

a. **Triangular numbers**: 21, 28 The differences increase by 1 each time: $15 + 6 = 21$ and $21 + 7 = 28$.
b. **Cube numbers**: 125, 216 These are 1^3, 2^3, 3^3, 4^3, so the next terms are $5^3 = 125$ and $6^3 = 216$.
c. **Fibonacci sequence** (add the previous two terms): 13, 21 Each term is the sum of the two before it: $5 + 8 = 13$ and $8 + 13 = 21$.
d. **Prime numbers**: 13, 17 Each number has exactly two factors. After 11 the next primes are 13 and 17.

Answer to Question 7

a. Add 2 each time: pattern 4 has **9** and pattern 5 has **11** matches. Each new triangle needs 2 more matches, because it shares one side with the triangle before it.
b. Difference 2, so $2n + 1$: $2n + 1$. The common difference 2 gives $2n$, and the zero term is $3 - 2 = 1$. Check: $2 \times 3 + 1 = 7$.
c. $2 \times 25 + 1 =$ **51 matches**. Substitute $n = 25$ into the nth term. Pattern 25 needs 51 matches.
d. $2n + 1 \leq 100$, so $n \leq 49.5$: pattern **49** uses 99 matches, leaving **1** match. Pattern 50 would need 101 matches, which is too many. So the biggest pattern is pattern 49, with 1 match left over.

Answer to Question 8

a. £65, £80, £95 She adds £15 each week: $50 + 15 = 65$, $65 + 15 = 80$, $80 + 15 = 95$.
b. $50 + 15n$ (an arithmetic sequence with first term 65 and difference 15, written $15n + 50$) The common difference is 15 and the zero term is the starting £50.
c. $50 + 15 \times 20 =$ **£350** Substitute $n = 20$. Ellie has £350 after 20 weeks.
d. $50 + 15n > 500$, $15n > 450$, $n > 30$, so **31 weeks**. After 30 weeks she has exactly £500, which is not more than £500. So she first has more than £500 after 31 weeks.

Answer to Question 9

a. $16 + 4 =$ **20**
b. The difference is 4, so $4n + \ldots$ and $8 - 4 = 4$. The nth term is $4n + 4$.
c. $4n + 4 = 60$, $4n = 56$, $n = 14$. White slabs $= 14^2 =$ **196**. So the patio has 14 white slabs on each side and 196 white slabs.
d. Each side has n slabs (not counting corners): $4 \times n$. The four corner slabs add 4 more: $4n + 4$.

Answer to Question 10

a. 2 tables: **10**. 3 tables: **14**. Each extra table adds 4 seats: $6 + 4 = 10$ and $10 + 4 = 14$.
b. Difference 4, first term 6, so $4n + 2$. The zero term is $6 - 4 = 2$. Check: $4 \times 3 + 2 = 14$.
c. $4n + 2 \geq 90$, $4n \geq 88$, $n \geq 22$: **22 tables** (exactly 90). Solve the inequality. Exactly 22 tables seat $4 \times 22 + 2 = 90$ people.
d. $4 \times 12 + 2 =$ **50 seats**. Substitute $n = 12$ into the nth term. Twelve tables seat 50 people.

Answer to Question 11

a. 100×2^n The number starts at 100 and is multiplied by 2 every hour, so after n hours it has been multiplied by 2^n.
b. $100 \times 256 =$ **25 600** Substitute $n = 8$: $2^8 = 256$. There are 25 600 bacteria after 8 hours.
c. $100 \times 2^n > 10^6$, $2^n > 10\,000$. $2^{13} = 8192$ and $2^{14} = 16\,384$. So **14 hours**. Divide both sides by 100, then try powers of 2. After 13 hours there are only 819 200 bacteria, so the number first passes 1 million after 14 hours.
d. It is **geometric** because each term is multiplied by the same number, 2. An arithmetic sequence would add the same number each time instead.

Answer to Question 12

a. £1050, £1102.50, £1157.63 Each year the amount is multiplied by 1.05: $1000 \times 1.05 = 1050$, $1050 \times 1.05 = 1102.50$, and $1102.50 \times 1.05 = 1157.625$, which rounds to £1157.63.

b. **1.05** A 5% increase means each term is 105% of the term before, so the common ratio is 1.05.

c. $1000 \times 1.05^5 =$ **£1276.28** Multiply £1000 by 1.05 five times, which is $1.05^5 = 1.27628\ldots$. Sam has £1276.28 after 5 years.

d. $1.05^8 = 1.477$, $1.05^9 = 1.551$. So **9 years**. Try powers of 1.05 until the amount first passes £1500. After 8 years Sam has about £1477, which is not enough, so it takes 9 years.

Answer to Question 13

a. **6** tins The row number tells us how many tins are in that row, so the 6th row has 6 tins.

b. $1 + 2 + 3 + 4 + 5 + 6 =$ **21** Add the tins in all six rows. This total is the 6th triangular number, 21.

c. $\dfrac{10 \times 11}{2} =$ **55** Substitute $n = 10$ into the formula. There are 55 tins in 10 rows.

d. $n = 19$: $\dfrac{19 \times 20}{2} = 190$; $n = 20$: $210 > 200$. So **19 rows** with **10** tins left over. Try values of n in the formula: 19 rows use 190 tins and 20 rows would need 210. So the shop makes 19 full rows with $200 - 190 = 10$ tins left over.

Answer to Question 14

a. $12, 14, 16, 18, 20$: **20 seats**. Add 2 seats for each row: the rows have 12, 14, 16, 18 and 20 seats. Row 5 has 20 seats.

b. Difference 2, $12 - 2 = 10$: $2n + 10$. The common difference is 2, and the zero term is $12 - 2 = 10$. Check: $2 \times 5 + 10 = 20$.

c. $2 \times 20 + 10 =$ **50 seats**. Substitute $n = 20$ into the nth term. The back row has 50 seats.

d. $12 + 14 + 16 + 18 + 20 =$ **80 seats**. Add the seats in the first 5 rows. There are 80 seats in the first 5 rows.

Answer to Question 15

a. Differences 6, 10, 14; second difference 4, so $2n^2$. Subtract: $3 - 2 = 1$, $9 - 8 = 1$, $19 - 18 = 1$. $2n^2 + 1$

b. Differences 5, 7, 9; second difference 2, so n^2. Subtract: $1, 3, 5$ which is $2n - 1$.

$n^2 + 2n - 1$

c. $2 \times 400 + 1 =$ **801** Substitute $n = 20$ into $2n^2 + 1$: $20^2 = 400$.

d. $n^2 + 2n - 1 = 62$, $n^2 + 2n - 63 = 0$, $(n + 9)(n - 7) = 0$, so $n = 7$: **yes, the 7th term** $(49 + 14 - 1 = 62)$.

Answer to Question 16

a. $4n + 3 = 4n + 2 + 1 = 2(2n + 1) + 1$, which is 2 times a whole number plus 1: **odd**.

b. $1 = 1^2$, $1 + 3 = 4 = 2^2$, $1 + 3 + 5 = 9 = 3^2$, $1 + 3 + 5 + 7 = 16 = 4^2$ ✓

c. $(n + 1)^2 - n^2 = 2n + 1$, which is **odd**.

d. $6n - 1 = 6(n - 1) + 5$, so the remainder after dividing by 6 is 5. It leaves remainder 2 when divided by 3 $(6n - 1 = 3(2n - 1) + 2)$, so it is never divisible by 3.

Answer to Question 17

a. 3rd: $a + b$. 4th: $a + 2b$. 5th: $2a + 3b$. Each new term is the sum of the two before it, so collect the a and b terms each time.

b. $a + 2b = 17$ and $2a + 3b = 27$. From the first, $a = 17 - 2b$. Then $34 - 4b + 3b = 27$, so $b = 7$ and $a = 3$. Solve the simultaneous equations by substitution. Check: $3 + 14 = 17$ and $6 + 21 = 27$.

c. **3, 7, 10, 17, 27, 44** Start with $a = 3$ and $b = 7$ and keep adding the last two terms.

d. 7th: $27 + 44 = 71$, 8th: $44 + 71 =$ **115**. Continue the sequence two more steps. The 8th term is 115.

Answer to Question 18

a. $ar = 12$ and $ar^4 = 96$. The nth term of a geometric sequence is ar^{n-1}, so the 2nd term is ar and the 5th term is ar^4.

b. $\frac{ar^4}{ar} = r^3 = \frac{96}{12} = 8$, so $r =$ **2**. Dividing cancels a, leaving $r^3 = 8$. The cube root of 8 is 2.

c. $a = 12 \div 2 =$ **6**. Terms: **6, 12, 24, 48, 96**. Divide the second term by the ratio to get the first term, then keep doubling.

d. $6 \times 2^9 = 6 \times 512 =$ **3072**. The 10th term is ar^9. The 10th term is 3072.

Answer to Question 19

a. There are 50 pairs each adding to 101: $50 \times 101 =$ **5050**. Pairing the first and last

numbers always gives 101, and 100 numbers make 50 pairs. The sum is 5050.

b. $3 \times 20 + 2 = \mathbf{62}$ Substitute $n = 20$ into $3n + 2$. The 20th term is 62.

c. $S_{20} = \frac{20}{2}(5 + 62) = 10 \times 67 = \mathbf{670}$ Use the formula with first term 5 and last term 62. The sum of the first 20 terms is 670.

d. $S_n = \frac{n}{2}(5 + 3n + 2) = \frac{n}{2}(3n + 7)$. $S_{25} = \frac{25}{2} \times 82 = 1025 > 1000$ and $S_{24} = 12 \times 79 = 948$. So **25 terms**. Try values of n in the formula for S_n: 24 terms give 948 and 25 terms give 1025. So 25 terms are needed.

Answer to Question 20

a. First differences 7, 11, 15; second differences 4, 4. Subtract each term from the next: $12 - 5 = 7$, $23 - 12 = 11$, $38 - 23 = 15$. The second differences are constant, so the sequence is quadratic.

b. Second difference 4, so $2n^2$. Remainders: $5 - 2 = 3$, $12 - 8 = 4$, $23 - 18 = 5$, which is $n + 2$. So $2n^2 + n + 2$. Half of the second difference gives the coefficient of n^2. Then the remainders 3, 4, 5 form the linear part $n + 2$.

c. $2 \times 100 + 10 + 2 = \mathbf{212}$ Substitute $n = 10$ into $2n^2 + n + 2$. Pattern 10 has 212 dots.

d. $n = 22$: $968 + 22 + 2 = 992$. $n = 23$: $1058 + 23 + 2 = 1083$. So **pattern 23**. Try values of n close to $\sqrt{500} \approx 22$. Pattern 22 has 992 dots and pattern 23 has 1083, so the first pattern with more than 1000 dots is pattern 23.