Lesson - Ratio, proportion and percentage change

Year: Year 10

Ratio and proportion appear on every GCSE paper, and they carry more marks than almost any other strand. In this chapter you share quantities in a ratio, use scale, compare best buys, work with percentage increases and decreases, reverse percentages and calculate simple interest. These topics are on both the Foundation and Higher tiers.

Starter activity

A recipe for 4 people uses 200 g of flour, 100 g of sugar and 50 g of butter. Write the quantities as a ratio in its simplest form. How would you adjust the recipe for 6 people, and for 10? What stays the same when you change the number of people, and what changes? Now suppose you only have 60 g of butter. How many people can you cook for?

Writing and simplifying ratios

Definition: ratio

A **ratio** compares quantities of the same kind. $3 : 5$ means 3 parts for every 5 parts.

Rule: simplifying ratios

A ratio compares quantities of the same kind. $a : b$ is simplified by dividing both parts by their highest common factor: $24 : 36 = 2 : 3$. If the parts are decimals or fractions, multiply to make them whole numbers first: $0.5 : 2 = 1 : 4$. To write a ratio in the form $1 : n$, divide both parts by the first: $5 : 2 = 1 : 0.4$.

Sharing in a ratio

Method: sharing in a ratio

1. Add the parts to get the total number of parts.
2. Divide the amount by the total number of parts to find the value of one part.
3. Multiply by each share.

Worked example 1

Share £120 in the ratio $3 : 5$: $3 + 5 = 8$ parts; $120 \div 8 = 15$; shares $3 \times 15 = £45$ and $5 \times 15 = £75$. Check they add to the total. With three parts, $2 : 3 : 5$ of 200: $200 \div 10 = 20$, giving 40, 60 and 100.

Worked example 2

Difference given. Two numbers are in the ratio $4 : 7$ and differ by 36. The difference is 3 parts, so one part is 12 and the numbers are 48 and 84.

Worked example 3

One share given. Boys to girls is $2 : 3$ and there are 12 boys. One part is $12 \div 2 = 6$, so girls $= 3 \times 6 = 18$.

Direct proportion and the unitary method

Definition: direct proportion

Two quantities are in **direct proportion** if they increase at the same rate: when one doubles, so does the other.

Rule: the unitary method

Two quantities are in direct proportion if they increase at the same rate: when one doubles, so does the other. Find the value of one unit, then scale up. A recipe for 4 people uses 300 g of flour, so 1 person needs 75 g and 10 people need 750 g. 8 apples cost £2.40, so 1 costs 30p and 12 cost £3.60.

Best buys

Method: finding the best buy

Compare the price per unit (per gram, per 100 g or per litre) or the amount per pound. 500 g for £1.60 is 32p per 100 g; 750 g for £2.25 is 30p per 100 g, so the larger pack is the better value. Always give a conclusion.

Worked example 4

Pack A is 500 g for £1.60. Pack B is 750 g for £2.25. Which is the better value?

Pack A: $160 \div 5 = 32$p per 100 g. Pack B: $225 \div 7.5 = 30$p per 100 g. Pack B is cheaper per 100 g, so Pack B is the better buy.

Scale and maps

Rule: scale and maps

A scale of $1:50\,000$ means 1 cm on the map is $50\,000$ cm in real life, which is 500 m. So 3 cm on the map is 1.5 km. To convert, multiply by the scale for map to real, divide for real to map. Remember to convert cm to m or km at the end: $100\,000$ cm $= 1$ km. A model car at $1:24$ with length 18 cm represents $18 \times 24 = 432$ cm $= 4.32$ m. Currency conversions use the same idea: if *£1=€1.15* then £240 is *240\times1.15=€276*.

Percentage change

Rule: percentage change and reverse percentages

Use multipliers: $\times 1.2$ for +20% and $\times 0.85$ for -15%. Percentage change $= \dfrac{\text{change}}{\text{original}} \times 100$. A profit made when buying for £40 and selling for £52 is $\frac{12}{40} \times 100 = 30\%$.

Reverse percentages. Divide by the multiplier. After a 15% reduction the price is £204, so the original is $204 \div 0.85 = £240$. If the new price is 120% of the original, then the original is the new price divided by 1.2. A fall of 15% to 8500 means the original was $8500 \div 0.85 = 10\,000$.

Repeated change. A £500 amount increased by 4% for two years: $500 \times 1.04 \times 1.04 = £540.80$. Multiply the multipliers; do not add the percentages.

Simple interest

Definition: simple interest

Simple interest is the same amount of interest every year, worked out on the original sum (the principal).

Rule: simple interest

Simple interest is the same amount each year: $I = P \times \frac{r}{100} \times t$. £800 at 3% per year for 4 years earns $800 \times 0.03 \times 4 = £96$. The total is the principal plus interest. VAT of 20% on a £45 price makes it $45 \times 1.2 = £54$.

Common mistakes

Watch out

- Dividing the total by the number of shares in the ratio rather than by the number of parts.
- Giving shares that do not add to the total.
- Calculating a percentage change from the new value instead of the original.
- Reversing a percentage by taking off the percentage instead of dividing by the multiplier.

- Forgetting units in scale problems: convert cm to m or km.

Ratio, proportion and percentage together

Ratio, proportion and percentage are three ways to compare quantities, and GCSE questions mix them freely. A ratio compares parts with parts, so $3:5$ means that for every 3 of the first there are 5 of the second. A fraction compares a part with the whole, so the same ratio means $\frac{3}{8}$ and $\frac{5}{8}$ of the total. Proportion describes how two quantities change together: in direct proportion they increase at the same rate, and in inverse proportion one increases as the other decreases. The most reliable method is the unitary method: find the value of one part or one unit first and then scale up. For sharing, add the parts, divide the total by the number of parts, and multiply by each share. Always check that the shares add up to the total.

Worked example 8: increasing in a ratio

A photo is 12 cm wide and is enlarged in the ratio $3:5$. Find the new width and the percentage increase [3 marks].

The scale factor is $\frac{5}{3} = 1.667$. New width $= 12 \times \frac{5}{3} = 20$ cm. The increase is 8 cm, which is $\frac{8}{12} = 66.7\%$. The area is increased by a factor $\left(\frac{5}{3}\right)^2 = \frac{25}{9}$, or about 178%, which is much more than the 67% increase in each length.

Worked example 9: reverse percentage in a ratio problem

Dev and Eli share some money in the ratio $5:3$. Dev gets £60 more than Eli. How much did each get? [3 marks]

The difference in parts is $5 - 3 = 2$ parts, which is £60, so 1 part is £30. Dev gets $5 \times 30 = £150$ and Eli gets $3 \times 30 = £90$. Check: $150 - 90 = 60$ ✓ and $150:90 = 5:3$ ✓. The total was £240.

Worked example 10: direct and inverse proportion together

4 machines make 600 bottles in 3 hours. How long do 6 machines take to make 1000 bottles? [4 marks]

One machine makes $600 \div 4 = 150$ bottles in 3 hours, which is 50 bottles per hour. Six machines make $6 \times 50 = 300$ bottles per hour. Time for 1000 bottles $= 1000 \div 300 = 3.33$ hours, or 3 hours 20 minutes. Combining proportions step by step avoids confusion about which direction to scale.

Worked example 11: percentage of a ratio share

Of 240 pupils, the ratio of boys to girls is $7:5$. 40% of the boys and 60% of

the girls play football. How many play football? [4 marks]

12 parts $= 240$, so 1 part $= 20$. Boys $= 140$, girls $= 100$. Boys playing: $0.4 \times 140 = 56$. Girls playing: $0.6 \times 100 = 60$. Total $= 116$, which is $\frac{116}{240} = 48.3\%$ of the pupils.

Note: units in ratios

A ratio is only meaningful if both quantities are in the same unit. Convert first: $50\text{ cm} : 2\text{ m}$ becomes $50 : 200 = 1 : 4$. A ratio has no units in its final form, but a scale such as $1 : 25\,000$ is used for lengths, and areas scale with the *square* of the length scale.

Watch out: parts and shares

A very common mistake is to divide the total by one of the ratio numbers instead of by the sum of the parts: to share £240 in the ratio $3 : 5$, divide by 8, not by 3 or 5. In percentage change, divide by the original value, never by the new one.

Did you know?

Ratios have been used by artists and architects for thousands of years. The ancient Greeks believed that the most beautiful rectangle has sides in the ratio of about $1 : 1.618$, the golden ratio. The Parthenon in Athens and many Renaissance paintings are said to use it. The A-series of paper sizes used in the UK, such as A4, is based on a different ratio, $1 : \sqrt{2}$, chosen so that when you cut a sheet in half, the two halves have the same shape as the original. This is why an A4 photocopy enlarged to 141% fits exactly onto A3. Ratios are also used in cooking, in mixing concrete, in maps and in medicine, where the strength of a dose depends on the ratio of a drug to a patient's body mass.

Otto's tip

Always find one part first. Write the total number of parts next to your working and check that your shares add up to the original amount. In percentage questions write the multiplier before you start.