Answers - Ratio, proportion and percentage change

Year: Year 10

Answer to Question 1

a. Divide by 12: $2:3$ 12 is the HCF of 24 and 36, so dividing both parts by 12 gives the simplest form $2:3$.
b. Multiply by 2 then divide by 3: $3:9=1:3$ First multiply by 2 to remove the decimals, giving $3:9$, then divide by 3. The simplest form is $1:3$.
c. Multiply by 4: $2:3$, so $2:3$ Multiplying both parts by 4, the common denominator, removes the fractions. The simplest form is $2:3$.
d. $1.5\text{ kg}=1500\text{ g}$, so $250:1500=1:6$ Both parts must be in the same units before simplifying, so change kilograms into grams, then divide by 250. The simplest form is $1:6$.

Answer to Question 2

a. $3+5=8$ parts, $240\div 8=30$. Shares: $3\times 30=$ **£90** and $5\times 30=$ **£150**. Add the parts to find the total number of parts, then find the value of one part. Check: $90+150=240$.
b. $2+5+1=8$ parts, $56\div 8=7$. Shares: **14, 35 and 7**. Check: $14+35+7=56$.
c. 4 parts, $120\text{p}\div 4=30\text{p}$. Shares: **30p and 90p**. Change £1.20 into 120p to make the division easier. Check: $30+90=120$.
d. The larger share is $\frac{5}{8}$ of the total. The larger share is 5 of the 8 equal parts.

Answer to Question 3

a. 3 parts $=21$, so 1 part $=7$. Blue $=7\times 7=$ **49**. Find the value of one part first, then multiply by 7. There are 49 blue counters.
b. $21+49=$ **70** (or 10×7). The total is $3+7=10$ parts. There are 70 counters altogether.
c. 1 part $=45\div 5=9$, girls $=6\times 9=$ **54**. Five parts represent the 45 boys, so one part is 9. There are 54 girls.
d. $4+3=7$ parts, $56\div 7=8$, so pears $=3\times 8=$ **24**. Add the parts to find the total number of parts, then find one part. Check: $32+24=56$.

Answer to Question 4

a. 1 apple $= 39$p, so 9 apples $=$ **£3.51** Divide £2.34 by 6 to find the cost of one apple, then multiply by 9.
b. 1 pencil $= 25$p, so 20 pencils $=$ **£5.00** Divide 300p by 12 to find the cost of one pencil, then multiply by 20.
c. 1 person $= 75$ g, so 5 people $=$ **375 g** Divide 600 g by 8 for one person, then multiply by 5.
d. 1 litre covers 8 m², so 22 litres cover **176 m²** Divide 120 m² by 15 to find what 1 litre covers, then multiply by 22.

Answer to Question 5

a. $180 \div 5 =$ **36p per 100 g** 500 g is 5 lots of 100 g, so we divide 180p by 5.
b. $255 \div 7.5 =$ **34p per 100 g** 750 g is 7.5 lots of 100 g, so we divide 255p by 7.5.
c. **Cereal B** is better value (34p against 36p per 100 g). The lower price per 100 g gives more cereal for the same money.
d. Juice A: $1.35 \div 1.5 = £0.90$ per litre. Juice B: $1.90 \div 2 = £0.95$ per litre. **Juice A** is better value. Divide each price by the number of litres. The lower price per litre is the better buy.

Answer to Question 6

a. $6.4 \times 25\,000 = 160\,000$ cm $= 1600$ m $=$ **1.6 km** Every centimetre on the map is 25 000 cm in real life. Then divide by 100 for metres and by 1000 for kilometres.
b. 3 km $= 300\,000$ cm. $300\,000 \div 25\,000 =$ **12 cm** Change kilometres into centimetres first, then divide by 25 000.
c. $25\,000$ cm $=$ **250 m** 25 000 cm divided by 100 is 250 m.
d. 1 cm² on the map is $250 \times 250 = 62\,500$ m². $4 \times 62\,500 = 250\,000$ m² $=$ **0.25 km²**. For areas, the scale is squared.

Answer to Question 7

a. 5 parts $= 15$ litres, so 1 part $= 3$ litres. Blue $= 9$ litres, yellow $= 6$ litres.
b. Blue $9 \times 8 = 72$. Yellow $6 \times 6.50 = 39$. Total $=$ **£111**.
c. $111 \div 15 =$ **£7.40 per litre**. Divide the total cost by the number of litres.
d. Blue is 3 parts so 1 part $= 1.5$ litres and the total is $5 \times 1.5 =$ **7.5 litres**. Blue is the limiting colour, so the most green paint he can make is 7.5 litres.

Answer to Question 8

a. 9 parts: $1260 \div 9 = 140$. Amir **£280**, Beth **£420**, Chen **£560**. Add the parts: $2 + 3 + 4 = 9$. Divide the prize by 9 to find one part. Check: $280 + 420 + 560 = 1260$.

b. $\frac{4}{9}$ Chen has 4 of the 9 equal parts, so his fraction is $\frac{4}{9}$.
c. $420 - 390 =$ **£30** Subtract the price of the bike from Beth's share. Beth has £30 left.
d. Yes. Chen has 4 parts and Amir has 2 parts, and $560 = 2 \times 280$. So Amir is correct.

Answer to Question 9

a. 2 parts $= 400$ g, so 1 part $= 200$ g. Flour $=$ **1000 g**, sugar $=$ **600 g**.
b. 10 parts $=$ **2000 g**.
c. $\frac{3}{10} =$ **30%** Sugar is 3 of the $5 + 3 + 2 = 10$ parts.
d. 1.5 kg $= 1500$ g, 1 part $= 150$ g: flour **750 g**, sugar **450 g**, butter **300 g**.

Answer to Question 10

a. $4.5 \times 20\,000 = 90\,000$ cm $=$ **900 m**. Every centimetre on the map is 20 000 cm in real life, and there are 100 cm in a metre.
b. $0.9 \text{ km} \div 5 = 0.18$ h $= 0.18 \times 60 =$ **10.8 minutes**. Change the distance into kilometres, then use time $=$ distance $\div$ speed. Multiply by 60 to change hours into minutes.
c. $90\,000 \div 9 = 10\,000$, so the scale is $1 : 10\,000$. The same real distance of 90 000 cm is shown by 9 cm, so 1 cm represents 10 000 cm.
d. 1 cm^2 represents $200 \times 200 = 40\,000$ m². $0.5 \text{ km}^2 = 500\,000$ m².
$500\,000 \div 40\,000 =$ **12.5 cm²**. The area scale is the square of the length scale.

Answer to Question 11

a. $250 \times 1.16 =$ **€290**
b. Left *=290-180=€110*. $110 \times 0.84 =$ **£92.40**
c. She spent €180, which is worth $180 \div 1.16 = £155.17$ at the first rate. So she has £92.40 and spent about £155.17, a total of 247.57. Her loss from the exchange is **£2.43** (to the nearest penny).
d. $\frac{2.43}{250} =$ **1.0%** (1 d.p.)

Answer to Question 12

a. $40 \times 1.15 =$ **£46** A 15% increase gives the multiplier 1.15.
b. $46 \times 0.85 =$ **£39.10** A 15% decrease gives the multiplier 0.85, applied to the new price.

c. £39.10 is less than £40. The 15% reduction is of the larger price £46. So the final price is 90p lower than the original price.
d. $1.15 \times 0.85 = 0.9775$, so a **2.3% decrease** (1 d.p.). The overall multiplier is less than 1, and $1 - 0.9775 = 0.0225$, so the price has fallen by 2.3%.

Answer to Question 13

a. $800 \times 0.035 =$ **£28** 3.5% of £800 is 0.035×800. Simple interest is the same every year because it is always worked out on the original £800.
b. $4 \times 28 =$ **£112**, total $= 800 + 112 =$ **£912** Multiply the yearly interest by 4, then add it to the original amount. Ravi has £912 after 4 years.
c. $100I = PRT$, so $T = \dfrac{100I}{PR}$ Multiply both sides by 100, then divide by PR.
d. $T = \dfrac{100 \times 210}{800 \times 3.5} = \dfrac{21\,000}{2800} =$ **7.5 years** Substitute $I = 210$, $P = 800$ and $R = 3.5$. Check: $28 \times 7.5 = 210$. It takes 7.5 years.

Answer to Question 14

a. 7 parts $= 1.4$, so 1 part $= 0.2$ m³: cement **0.2 m³**, sand **0.4 m³**, gravel **0.8 m³**.
b. Cement $2 \times 9 = 18$, sand $4 \times 4 = 16$, gravel $8 \times 5 = 40$. Total = **£74**.
c. $\dfrac{4}{7} =$ **57.1%**
d. Sand is 2 parts $= 0.5$, so 1 part $= 0.25$ and the total is $7 \times 0.25 =$ **1.75 m³**.

Answer to Question 15

a. $\dfrac{3k+6}{5k+6} = \dfrac{5}{7}$, so $7(3k+6) = 5(5k+6)$. In 6 years both ages increase by 6, so Maya will be $3k+6$ and her father $5k+6$. Cross-multiply to remove the fractions.
b. $21k + 42 = 25k + 30$, so $12 = 4k$ and $k =$ **3**. Expand both sides, then subtract $21k$ and 30 from both sides.
c. Maya **9**, her father **15**. Substitute $k = 3$: Maya is $3 \times 3 = 9$ and her father is $5 \times 3 = 15$.
d. In 6 years: $15 : 21 = 5 : 7$ ✓. In 6 years Maya will be 15 and her father 21, and $15 : 21$ simplifies to $5 : 7$ by dividing by 3.

Answer to Question 16

a. $A : B = 2 : 3 = 8 : 12$ and $B : C = 4 : 5 = 12 : 15$. 12 is the LCM of 3 and 4, so we multiply the first ratio by 4 and the second ratio by 3.
b. $A : B : C = 8 : 12 : 15$. Now B is 12 in both ratios, so the ratios can be joined.

c. 35 parts $= 175$, so 1 part $= 5$: A **£40**, B **£60**, C **£75**. Add the parts: $8 + 12 + 15 = 35$. Check: $40 + 60 + 75 = 175$.
d. $\frac{15}{35} = \frac{3}{7}$ C collected 15 of the 35 parts, which simplifies to $\frac{3}{7}$.

Answer to Question 17

a. $1.1 \times 1.2 = 1.32$, so **32% increase**. Multiply the two multipliers. The result is not 30%, because the 20% is worked out on the larger amount.
b. $1200 \times 1.04 \times 1.02 =$ **£1272.96**. Multiplier 1.0608, so **6.08%** increase. Use the multipliers 1.04 and 1.02 one after the other.
c. $\frac{1}{1.25} = 0.8$, so a **20% decrease**. To reverse a multiplier we divide by it, so the new multiplier is $1 \div 1.25 = 0.8$.
d. $0.92^3 = 0.7787$ and $0.92^4 = 0.7164$. After **4 years** it is below 75%. Keep multiplying by 0.92 until the result first drops below 0.75. It takes 4 years.

Answer to Question 18

a. A: $0.2 \times 8 = 1.6$ kg. B: $0.45 \times 12 = 5.4$ kg. Multiply each mass by its percentage of copper.
b. Copper $= 1.6 + 5.4 = 7$ kg. Total $= 8 + 12 = 20$ kg. Add the copper from both alloys and the total masses.
c. $\frac{7}{20} =$ **35%** Divide the mass of copper by the total mass and multiply by 100.
d. Let b kg be added: $1.6 + 0.45b = 0.4(8 + b) = 3.2 + 0.4b$, so $0.05b = 1.6$ and $b =$ **32 kg**. Check: $1.6 + 0.45 \times 32 = 16$ kg of copper in 40 kg, which is 40%.

Answer to Question 19

a. $3 \times 8 + 2 \times 13 = 24 + 26 =$ **£50** Multiply each mass by its price per kilogram, then add. The mixture costs £50.
b. $50 \div 5 =$ **£10 per kg** Divide the total cost by the total mass of $3 + 2 = 5$ kg. The blend costs £10 per kg.
c. $10 \times 1.3 =$ **£13 per kg** A 30% profit gives the multiplier 1.3. The blend should be sold at £13 per kg.
d. $0.45 \times 13 = 5.85$, so **£5.85**. Change 450 g into 0.45 kg before multiplying. She pays £5.85.

Answer to Question 20

a. Juice $= 2k$ and water $= 3k$. The ratio $2 : 3$ means there are 2 equal parts of juice

and 3 equal parts of water, each of size k litres.

b. $\frac{2k}{3k+10} = \frac{1}{2}$, so $4k = 3k + 10$ and $k = 10$. Adding water changes only the water, so the juice stays $2k$. Cross-multiply, then subtract $3k$ from both sides.

c. Juice $= 2 \times 10 =$ **20 litres** (water 30 litres). Substitute $k = 10$: there were 20 litres of juice.

d. $20 + 30 + 10 =$ **60 litres**, in the ratio $20 : 40 = 1 : 2$ ✓. The final drink has 20 litres of juice and 40 litres of water.