Worksheet - Quadratic equations and graphs

Year: Year 10 · 20

Show your working.

Question 1

Question 1 – Solving by factorising *(Section A · Skills practice)*

Non-calculator. Solve each quadratic equation by factorising.

a. $x^2 + 5x + 6 = 0$
b. $x^2 - 2x - 15 = 0$
c. $x^2 - 9x = 0$
d. $x^2 - 49 = 0$

Question 2

Question 2 – Rearranging before factorising *(Section A · Skills practice)*

Rearrange each equation so that one side is zero, then factorise and solve.

a. $x^2 = 5x + 14$
b. $x^2 + x = 12$
c. $x(x - 3) = 18$
d. $(x + 2)^2 = 25$

Question 3

Question 3 – Plotting a quadratic graph *(Section A · Skills practice)*

Complete the table of values for $y = x^2 - 2x - 3$ and use it to answer the questions.

x	-2	-1	0	1	2	3	4
y	?	?	?	?	?	?	?

a. Complete the table.
b. Write down the roots of $x^2 - 2x - 3 = 0$ from your table.
c. Write down the coordinates of the turning point.
d. Write down the equation of the line of symmetry.

Question 4

Question 4 – Features of a parabola *(Section A · Skills practice)*

The graph of $y = x^2 - 6x + 5$ is a U-shaped parabola.

a. Find where it crosses the x-axis by factorising.
b. Find where it crosses the y-axis.
c. Use the symmetry of the roots to find the equation of the line of symmetry.
d. Find the coordinates of the turning point and say whether it is a minimum or maximum.

Question 5

Question 5 – Sketching quadratics *(Section A · Skills practice)*

Sketch each curve, showing where it crosses the axes. Describe the shape and say whether the turning point is a maximum or a minimum.

a. $y = x^2 - 4x$
b. $y = 4 - x^2$
c. $y = (x - 2)(x + 3)$
d. $y = -x^2 + 2x + 3$

Question 6

Question 6 – Solving by drawing a graph *(Section A · Skills practice)*

Here is the table of values for $y = x^2 - 2x - 3$ from $x = -2$ to $x = 4$:
$y = 5, 0, -3, -4, -3, 0, 5$. Use the graph (draw it if you can) to solve each equation to 1 decimal place.

a. $x^2 - 2x - 3 = 2$ (draw the line $y = 2$)
b. $x^2 - 2x - 3 = -3$
c. $x^2 - 2x - 3 = 0$
d. What does the equation $x^2 - 2x - 3 = -5$ tell you about the graph? Explain.

Question 7

Question 7 – The thrown ball *(Section B · Real-life practice)*

A ball is thrown upwards from the ground. Its height h metres after t seconds is $h = 20t - 5t^2$. Calculator allowed.

t (s)	0	1	2	3	4
h (m)	0	15	20	15	0

a. Check the table value at $t = 3$.
b. What is the maximum height and when does it happen?
c. When is the ball on the ground? Solve $20t - 5t^2 = 0$.
d. For how long is the ball higher than 15 m? Solve $20t - 5t^2 = 15$.

Question 8

Question 8 – The pen for the rabbits *(Section B · Real-life practice)*

Ola has 24 m of fencing to make a rectangular rabbit run against a wall. A different arrangement uses all 24 m for a rectangle with length x metres. The width is $12 - x$ metres.

a. Write the area A in terms of x.
b. Solve $x(12 - x) = 32$ to find the possible lengths when the area is 32 m².
c. Find the value of x that gives the greatest area and find that area.
d. What shape gives the greatest area? Explain.

Question 9

Question 9 – The footbridge arch *(Section B · Real-life practice)*

The height h metres of a footbridge arch at a distance x metres from one end is $h = \dfrac{x(8 - x)}{4}$ for $0 \le x \le 8$.

a. What is the height at the ends of the arch?
b. What is the height at the highest point, and where is it?
c. At what two distances is the arch 3 metres high?
d. A boat 3 m high must pass under the arch. How wide is the gap with a clearance of at least 3 m?

Question 10

Question 10 – Garden dimensions *(Section B · Real-life practice)*

A rectangular lawn has width x metres and length $(x + 3)$ metres. Its area is 40 m².

a. Show that $x^2 + 3x - 40 = 0$.
b. Solve the equation by factorising.
c. Explain why one solution is rejected.
d. Find the perimeter of the lawn.

Question 11

Question 11 – Market stall profit *(Section B · Real-life practice)*

A stall sells scarves. The profit P pounds a day when the price is x pounds is $P = -x^2 + 10x - 16$.

a. Find the prices at which the stall breaks even ($P = 0$).
b. Find the price that gives the greatest profit, and the profit.
c. Find the profit when $x = 3$.
d. Describe the shape of the graph and explain what the roots mean.

Question 12

Question 12 – Right-angled triangle *(Section B · Real-life practice)*

A ramp forms a right-angled triangle with the ground. The shorter side is x metres, the longer side is $(x + 7)$ metres and the sloping side is 13 metres.

a. Use Pythagoras to show that $x^2 + 7x - 60 = 0$.
b. Solve by factorising.
c. Find the three side lengths.
d. Verify that they form a right-angled triangle.

Question 13

Question 13 – Jumping off the diving board *(Section B · Real-life practice)*

A diver steps off a board 10 m above the water. Her height above the water after t seconds is $h = 10 - 5t^2$ (metres), ignoring air resistance. Calculator allowed.

a. Work out her height after 1 second.
b. When does she hit the water? Give your answer to 2 decimal places.
c. Sketch the graph for $0 \leq t \leq 1.5$ and say what shape it is.
d. When is she 5 m above the water?

Question 14

Question 14 – The goal kick *(Section B · Real-life practice)*

A goalkeeper kicks a football. Its height in metres after it has travelled x metres along the ground is $y = \dfrac{x(50 - x)}{50}$.

a. How far does the ball travel before it lands?
b. What is the greatest height, and how far along is it reached?
c. Find the height when $x = 10$.
d. A defender can head the ball at heights below 3 m. At what two distances is the ball exactly 3 m high? Give answers to 1 decimal place.

Question 15

Question 15 – Number problems *(Section C · Problem solving)*

Form a quadratic equation for each problem and solve it by factorising.

a. The product of two consecutive whole numbers is 156. Find them.
b. Two numbers differ by 5 and their product is 84. Find them.
c. A number squared plus the number itself equals 90. Find the positive number.
d. Show that no number can be added to its square to give -1.

Question 16

Question 16 – Higher: factorising when a is not 1 *(Section C · Problem solving)*

Higher tier. Solve each equation by factorising. Show how you split the middle term.

a. $2x^2 + 5x - 12 = 0$
b. $3x^2 - 11x - 4 = 0$
c. $6x^2 - 7x - 3 = 0$
d. Find where $y = 2x^2 + 5x - 12$ crosses the axes.

Question 17

Question 17 – Higher: completing the square *(Section C · Problem solving)*

Higher tier. Use $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 + c - \frac{b^2}{4}$.

a. Write $x^2 + 6x + 1$ in the form $(x + a)^2 + b$.
b. Write down the turning point of $y = x^2 + 6x + 1$.
c. Solve $x^2 + 6x + 1 = 0$, leaving your answer in surd form.
d. Write $x^2 - 4x - 3$ in the form $(x + a)^2 + b$ and solve $x^2 - 4x - 3 = 0$ to 2 decimal places.

Question 18

Question 18 – Higher: the quadratic formula *(Section C · Problem solving)*

Higher tier. The solutions of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Give answers to 2 decimal places.

a. Solve $2x^2 + 3x - 4 = 0$.
b. Solve $x^2 - 5x + 3 = 0$.
c. Work out the discriminant $b^2 - 4ac$ of $3x^2 + 2x + 1 = 0$ and say what it tells you.
d. For what values of k does $x^2 + 4x + k = 0$ have two different real roots?

Question 19

Question 19 – Higher: field problem *(Section C · Problem solving)*

Higher tier. A rectangular field is 5 m longer than it is wide, and has an area of 120 m². Calculator allowed.

a. Let the width be w metres. Show that $w^2 + 5w - 120 = 0$.
b. Solve using the formula, giving answers to 2 decimal places.
c. State which answer is valid and find the length of the field.
d. Find the perimeter of the field to 1 decimal place.

Question 20

Question 20 – Challenge: when does the graph touch the axis? *(Section D · Challenge)*

Consider $y = x^2 - 4x + k$ where k is a number.

a. Show that the discriminant is $16 - 4k$.
b. For what value of k does the graph touch the x-axis at just one point? Find that point.
c. For what values of k does the graph not cross the x-axis at all?
d. Find the roots of $x^2 - 7x + 10 = 0$ and check that their sum is 7 and their product is 10.