Answers - Quadratic equations and graphs

Year: Year 10

Answer to Question 1

a. $(x+2)(x+3)=0$, so $x=-2$ **or** $x=-3$ We need two numbers with product 6 and sum 5: 2 and 3. If a product is zero, one of the factors must be zero, so $x+2=0$ or $x+3=0$.
b. $(x-5)(x+3)=0$, so $x=5$ **or** $x=-3$ We need product -15 and sum -2: -5 and 3. Check: $25-10-15=0$.
c. $x(x-9)=0$, so $x=0$ **or** $x=9$ Take out the common factor x. Do not divide both sides by x, because that would lose the solution $x=0$.
d. $(x-7)(x+7)=0$, so $x=7$ **or** $x=-7$ This is a difference of two squares, so there are two solutions, one positive and one negative.

Answer to Question 2

a. $x^2-5x-14=0$, $(x-7)(x+2)=0$, so $x=7$ **or** $x=-2$ Subtract $5x$ and 14 from both sides so that one side is zero. Check: $49=35+14$.
b. $x^2+x-12=0$, $(x+4)(x-3)=0$, so $x=-4$ **or** $x=3$ Subtract 12 from both sides first. Check: $9+3=12$.
c. $x^2-3x-18=0$, $(x-6)(x+3)=0$, so $x=6$ **or** $x=-3$ Expand the bracket and subtract 18. Check: $6\times3=18$ and $-3\times(-6)=18$.
d. $x+2=\pm5$, so $x=3$ **or** $x=-7$ Take the square root of both sides, remembering both the positive and the negative root. Check: $5^2=25$ and $(-5)^2=25$.

Answer to Question 3

a. $x=-2$: $4+4-3=5$; -1: $1+2-3=0$; 0: -3; 1: $1-2-3=-4$; 2: $4-4-3=-3$; 3: $9-6-3=0$; 4: $16-8-3=5$. So **5, 0, −3, −4, −3, 0, 5**.
b. The curve crosses the x-axis where $y=0$: $x=-1$ **and** $x=3$.
c. The lowest point is $(1,-4)$.
d. The line of symmetry passes through the turning point: $x=1$.

Answer to Question 4

a. $(x-1)(x-5)=0$, so $(1,0)$ **and** $(5,0)$. The curve meets the x-axis where $y=0$. We need product 5 and sum -6: -1 and -5.
b. When $x=0$, $y=$ **5**, so $(0,5)$. The y-intercept is the constant term, because every

other term is zero when $x = 0$.

c. Halfway between 1 and 5: $x = 3$. A parabola is symmetrical, so the line of symmetry is halfway between the two roots.
d. At $x = 3$: $y = 9 - 18 + 5 = -4$. The turning point is $(3, -4)$, a **minimum** (the coefficient of x^2 is positive). Substitute $x = 3$ into the equation to find the y-coordinate.

Answer to Question 5

a. Roots: $x(x - 4) = 0$ gives $(0, 0)$ and $(4, 0)$. U-shape with **minimum** at $(2, -4)$.
b. Roots $x = \pm 2$, y-intercept 4. An upside-down U (**maximum** at $(0, 4)$).
c. Roots $x = 2$ and $x = -3$, y-intercept -6. U-shape with a minimum at $x = -0.5$, $y = -6.25$.
d. $-(x^2 - 2x - 3) = -(x - 3)(x + 1)$, roots 3 and -1, y-intercept 3. Upside-down U with **maximum** at $(1, 4)$.

Answer to Question 6

a. The line $y = 2$ crosses the curve at about $x = -1.4$ **and** $x = 3.4$ (exact: $x = 1 \pm \sqrt{6}$).
b. At $y = -3$: $x = 0$ **and** $x = 2$. Read across from $y = -3$ on the y-axis to the curve and down to the x-axis.
c. At $y = 0$: $x = -1$ **and** $x = 3$. These are the roots, where the curve crosses the x-axis.
d. The lowest value of y is -4, so the line $y = -5$ never meets the graph: the equation has **no solutions**.

Answer to Question 7

a. $20 \times 3 - 5 \times 9 = 60 - 45 =$ **15 m** ✓
b. The turning point is at $t = 2$ (halfway between the zeros): $h = 40 - 20 =$ **20 m**.
c. $5t(4 - t) = 0$, so $t = 0$ **and** $t = 4$: it lands after 4 seconds.
d. $5t^2 - 20t + 15 = 0$, $t^2 - 4t + 3 = 0$, $(t - 1)(t - 3) = 0$, so $t = 1$ or 3. It is above 15 m for $3 - 1 = 2$ **seconds**.

Answer to Question 8

a. $A = x(12 - x) = 12x - x^2$.
b. $x^2 - 12x + 32 = 0$, $(x - 4)(x - 8) = 0$, so $x = 4$ **or** $x = 8$ (a 4×8 rectangle, the same shape).
c. By symmetry the maximum is halfway between 0 and 12, at $x = 6$, area

$= 6 \times 6 =$ **36 m²**.

d. A **square** ($6\ \text{m} \times 6\ \text{m}$) gives the greatest area for a fixed perimeter.

Answer to Question 9

a. At $x = 0$ and $x = 8$ the height is **0 m**.

b. By symmetry $x = 4$: $h = \dfrac{4 \times 4}{4} =$ **4 m**.

c. $x(8 - x) = 12$, $x^2 - 8x + 12 = 0$, $(x - 2)(x - 6) = 0$: $x = 2$ **and** $x = 6$.

d. The arch is at least 3 m high between $x = 2$ and $x = 6$, so the gap is **4 m wide**.

Answer to Question 10

a. Area $= x(x + 3) = 40$, so $x^2 + 3x = 40$ and $x^2 + 3x - 40 = 0$. The area of a rectangle is width times length.

b. $(x + 8)(x - 5) = 0$, so $x = -8$ or $x = 5$. We need two numbers with product -40 and sum 3: 8 and -5.

c. A length cannot be negative, so $x = 5$ only. So the width of the lawn is 5 m.

d. Width 5 m, length 8 m, perimeter $= 2(5 + 8) =$ **26 m**. Check: $5 \times 8 = 40$ m², so the perimeter of the lawn is 26 m.

Answer to Question 11

a. $x^2 - 10x + 16 = 0$, $(x - 2)(x - 8) = 0$: **£2 and £8**.

b. Halfway between 2 and 8: **£5**, with $P = -25 + 50 - 16 =$ **£9**.

c. $-9 + 30 - 16 =$ **£5**. Substitute $x = 3$ into the formula. The profit is £5.

d. An upside-down U. The roots are the prices at which the profit is zero: below £2 or above £8 the stall makes a loss.

Answer to Question 12

a. $x^2 + (x + 7)^2 = 169$, so $2x^2 + 14x + 49 = 169$, $2x^2 + 14x - 120 = 0$, and dividing by 2: $x^2 + 7x - 60 = 0$. The two shorter sides are x and $x + 7$, and the hypotenuse is 13. Expand $(x + 7)^2 = x^2 + 14x + 49$.

b. $(x + 12)(x - 5) = 0$, so $x = -12$ or $x = 5$. The length must be positive, so $x = 5$. We need product -60 and sum 7: 12 and -5.

c. **5 m, 12 m, 13 m**. The sides are $x = 5$, $x + 7 = 12$ and the hypotenuse 13.

d. $5^2 + 12^2 = 25 + 144 = 169 = 13^2$ ✓ By the converse of Pythagoras' theorem, the triangle has a right angle.

Answer to Question 13

a. $10 - 5 \times 1 = $ **5 m**.
b. $10 - 5t^2 = 0$, $t^2 = 2$, $t = \sqrt{2} = $ **1.41 s**. Set the height equal to zero, divide by 5 and take the positive square root, because time cannot be negative.
c. The graph is part of an upside-down parabola with $h = 10$ at $t = 0$, falling to 0 at $t = 1.41$.
d. $10 - 5t^2 = 5$, $t^2 = 1$, so $t = 1$ **s** (the same as part (a)).

Answer to Question 14

a. $y = 0$ when $x = 0$ or $x = 50$: the ball lands **50 m** away.
b. The maximum is halfway at $x = 25$: $y = \dfrac{25 \times 25}{50} = $ **12.5 m**.
c. $y = \dfrac{10 \times 40}{50} = $ **8 m**.
d. $x(50 - x) = 150$, $x^2 - 50x + 150 = 0$, $x = \dfrac{50 \pm \sqrt{2500 - 600}}{2} = \dfrac{50 \pm 43.59}{2}$, so $x = 3.2$ **m and** $x = 46.8$ **m**. So the ball is 3 m high at about 3.2 m and 46.8 m from the goalkeeper.

Answer to Question 15

a. $n(n + 1) = 156$, $n^2 + n - 156 = 0$, $(n - 12)(n + 13) = 0$, so the numbers are **12 and 13**.
b. $x(x + 5) = 84$, $x^2 + 5x - 84 = 0$, $(x - 7)(x + 12) = 0$. Positive: **7 and 12**. (Also -12 and -7.)
c. $x^2 + x - 90 = 0$, $(x + 10)(x - 9) = 0$, so the positive number is **9**.
d. $x^2 + x + 1 = 0$ has discriminant $1 - 4 = -3 < 0$, so there are **no real solutions**. A negative discriminant means the graph of $y = x^2 + x + 1$ never crosses the x-axis.

Answer to Question 16

a. $2x^2 + 8x - 3x - 12 = 2x(x + 4) - 3(x + 4) = (2x - 3)(x + 4) = 0$, so $x = 1.5$ **or** $x = -4$ Multiply $a \times c = 2 \times (-12) = -24$ and find two numbers with product -24 and sum 5: 8 and -3. Check: $(2x - 3)(x + 4) = 2x^2 + 5x - 12$.
b. $3x^2 - 12x + x - 4 = 3x(x - 4) + (x - 4) = (3x + 1)(x - 4) = 0$, so $x = -\frac{1}{3}$ **or** $x = 4$ Here $ac = -12$, and -12 and 1 have sum -11. Check: $(3x + 1)(x - 4) = 3x^2 - 11x - 4$.
c. $6x^2 - 9x + 2x - 3 = 3x(2x - 3) + (2x - 3) = (3x + 1)(2x - 3) = 0$, so $x = -\frac{1}{3}$ **or**

$x = 1.5$ Here $ac = -18$, and -9 and 2 have sum -7. Check: $(3x + 1)(2x - 3) = 6x^2 - 7x - 3$.

d. x-axis: $(1.5, 0)$ and $(-4, 0)$. y-axis: $x = 0$ gives $y = -12$, so $(0, -12)$. The x-intercepts are the solutions from part (a), and the y-intercept is the constant term.

Answer to Question 17

a. $(x + 3)^2 - 9 + 1 = (x + 3)^2 - 8$ Halve the coefficient of x to get 3, then subtract $3^2 = 9$ to keep the expression the same.

b. The minimum is when $(x + 3)^2 = 0$: $(-3, -8)$. A square is never negative, so y is smallest when the bracket is zero, at $x = -3$.

c. $(x + 3)^2 = 8$, $x + 3 = \pm\sqrt{8}$, so $x = -3 \pm 2\sqrt{2}$ Rearrange the completed square, take the square root of both sides and simplify $\sqrt{8} = 2\sqrt{2}$.

d. $(x - 2)^2 - 4 - 3 = (x - 2)^2 - 7$. Then $(x - 2)^2 = 7$, $x = 2 \pm \sqrt{7}$, so $x = 4.65$ **or** $x = -0.65$. Halve -4 to get -2, then subtract $(-2)^2 = 4$. Using $\sqrt{7} = 2.6458$, the solutions are 4.65 and -0.65.

Answer to Question 18

a. $x = \dfrac{-3 \pm \sqrt{9 + 32}}{4} = \dfrac{-3 \pm \sqrt{41}}{4}$, so $x = 0.85$ **or** $x = -2.35$ Here $a = 2$, $b = 3$ and $c = -4$, so $b^2 - 4ac = 9 + 32 = 41$. The solutions are 0.85 and -2.35.

b. $x = \dfrac{5 \pm \sqrt{25 - 12}}{2} = \dfrac{5 \pm \sqrt{13}}{2}$, so $x = 4.30$ **or** $x = 0.70$ Here $a = 1$, $b = -5$ and $c = 3$, so $-b = 5$ and $b^2 - 4ac = 13$. The solutions are 4.30 and 0.70.

c. $b^2 - 4ac = 4 - 12 = -8 < 0$, so there are **no real roots**. We cannot take the square root of a negative number, so the graph of $y = 3x^2 + 2x + 1$ does not cross the x-axis.

d. Two different real roots need $16 - 4k > 0$, so $k < 4$. Here $b^2 - 4ac = 4^2 - 4 \times 1 \times k$, and the discriminant must be positive for two different roots.

Answer to Question 19

a. Area $= w(w + 5) = 120$, so $w^2 + 5w - 120 = 0$.

b. $w = \dfrac{-5 \pm \sqrt{25 + 480}}{2} = \dfrac{-5 \pm \sqrt{505}}{2}$, so $w = 8.74$ or $w = -13.74$. Here $a = 1$, $b = 5$, $c = -120$, so $b^2 - 4ac = 25 + 480 = 505$.

c. The width must be positive: $w =$ **8.74 m**, length = **13.74 m**.

d. Perimeter $= 2(8.74 + 13.74) =$ **44.9 m** (to 1 d.p., using 44.96).

Answer to Question 20

a. $b^2 - 4ac = (-4)^2 - 4 \times 1 \times k = 16 - 4k$.
b. One repeated root needs $16 - 4k = 0$, so $k = 4$: $x^2 - 4x + 4 = (x - 2)^2$, touching at $(2, 0)$.
c. No real roots needs $16 - 4k < 0$, so $k > 4$. Divide both sides by -4 and reverse the inequality sign.
d. $(x - 2)(x - 5) = 0$: roots 2 and 5. Sum $= 7$ ✓, product $= 10$ ✓.