Answers - Linear equations and inequalities

Year: Year 10

Answer to Question 1

a. $3x = 18$, so $x = 6$ (check: $18 + 7 = 25$ ✓) Subtract 7 from both sides, then divide both sides by 3. The solution is $x = 6$.
b. $-2x = 8$, so $x = -4$ (check: $5 + 8 = 13$ ✓) Subtract 5 from both sides, then divide by -2; a positive divided by a negative is negative. The solution is $x = -4$.
c. $\frac{x}{4} = 5$, so $x = 20$ Add 3 to both sides, then multiply by 4 to undo the division. Check: $20 \div 4 - 3 = 2$. The solution is $x = 20$.
d. $x - 3 = 5$, so $x = 8$ (or $4x - 12 = 20$, $4x = 32$) Divide both sides by 4 first, then add 3. Check: $4 \times (8 - 3) = 20$. The solution is $x = 8$.

Answer to Question 2

a. $3x = 15$, so $x = 5$ Subtract $2x$ from both sides and add 3 to both sides. Check: $25 - 3 = 22$ and $10 + 12 = 22$. The solution is $x = 5$.
b. $16 = 4x$, so $x = 4$ The x coefficient is bigger on the right, so add $3x$ and add 9 to both sides. Check: $7 - 12 = -5$ and $4 - 9 = -5$. The solution is $x = 4$.
c. $8x - 4 = 3x + 21$, $5x = 25$, so $x = 5$ Expand both brackets first, then subtract $3x$ and add 4. Check: $4 \times 9 = 36$ and $3 \times 12 = 36$. The solution is $x = 5$.
d. $6 - 2x + 2 = 3x - 7$, $8 - 2x = 3x - 7$, $15 = 5x$, so $x = 3$ Be careful: $-2 \times (-1) = +2$. Then add $2x$ and 7 to both sides. Check: $6 - 4 = 2$ and $9 - 7 = 2$. The solution is $x = 3$.

Answer to Question 3

a. $x + 3 = 14$, so $x = 11$ Multiply both sides by 2 to clear the fraction, then subtract 3. Check: $\frac{14}{2} = 7$. The solution is $x = 11$.
b. $2x - 1 = 15$, $2x = 16$, so $x = 8$ Multiply both sides by 3, add 1, then divide by 2. Check: $\frac{15}{3} = 5$. The solution is $x = 8$.
c. Multiply by 6: $3x + 2x = 60$, $5x = 60$, so $x = 12$ The common denominator of 2 and 3 is 6, so every term is multiplied by 6. Check: $6 + 4 = 10$. The solution is $x = 12$.
d. Multiply by 4: $3x + 1 = 2(x + 9) = 2x + 18$, so $x = 17$ Multiply both sides by 4; the right side becomes $2(x + 9)$. Then subtract $2x$ and 1. Check: $\frac{52}{4} = 13$ and $\frac{26}{2} = 13$. The solution is $x = 17$.

Answer to Question 4

a. $3x > 15$, so $x > 5$ Solve it like an equation: add 4 to both sides, then divide by 3. The solution is $x > 5$.
b. $-2x \geq 4$, divide by -2 and reverse: $x \leq -2$ Subtract 5 from both sides. Dividing by a negative number reverses the sign, so $\geq$ becomes $\leq$. The solution is $x \leq -2$.
c. Subtract 1: $-4 < 2x \leq 8$. Divide by 2: $-2 < x \leq 4$ Do the same thing to all three parts of a double inequality. The solution is $-2 < x \leq 4$.
d. $4x - 4 < 3x + 2$, $x < 6$: $x < 6$ Expand the bracket, then subtract $3x$ and add 4 to both sides. The solution is $x < 6$.

Answer to Question 5

a. **−1, 0, 1, 2, 3, 4** (-2 is not included because the sign is $<$)
b. $3x < 18$, so $x < 6$. The largest integer is **5**.
c. $2x \geq 8$, so $x \geq 4$. The smallest integer is **4**.
d. Draw a number line, put a **filled (closed) circle** at -2 because -2 is included, and draw an arrow pointing to the left.

Answer to Question 6

a. $4x - 9 = 31$, $4x = 40$, so the number is **10**. Let the number be x. Add 9 to both sides, then divide by 4. Check: $4 \times 10 - 9 = 31$.
b. $x + (x + 3) + 4x = 63$, $6x + 3 = 63$, $6x = 60$, so Ben is **10** (sister 13, mother 40). Add the three ages and collect like terms. Check: $10 + 13 + 40 = 63$.
c. $n + (n + 1) + (n + 2) = 96$, $3n + 3 = 96$, $n = 31$: the numbers are **31, 32, 33**. Consecutive numbers differ by 1, so we call them n, $n + 1$ and $n + 2$. Check: $31 + 32 + 33 = 96$.
d. $2(x + 5) + 2x = 38$, $4x + 10 = 38$, $x = 7$: **7 cm by 12 cm**. The perimeter is two lengths plus two widths. Check: $2 \times 12 + 2 \times 7 = 38$.

Answer to Question 7

a. Plan A: $12 + 0.03m$. Plan B: $5 + 0.10m$.
b. $12 + 0.03m = 5 + 0.10m$, so $7 = 0.07m$ and $m =$ **100 minutes**. Subtract $0.03m$ and 5 from both sides, then divide by 0.07.
c. $12 + 0.03 \times 100 =$ **£15** (and $5 + 10 = 15$).
d. For 150 minutes: A costs $12 + 4.50 = £16.50$ and B costs $5 + 15 = £20$. **Plan A** is cheaper, because above 100 minutes the lower rate per minute outweighs the higher fee.

Answer to Question 8

a. $2(3x-2)+2(x+4)=60$, so $6x-4+2x+8=60$, so $8x+4=60$. The perimeter is two lengths plus two widths; expand the brackets and collect like terms.
b. $8x=56$, so $x=7$. Subtract 4 from both sides, then divide by 8.
c. Length $=3\times 7-2=$ **19 m**, width $=7+4=$ **11 m**, area $=19\times 11=$ **209 m²**. Substitute $x=7$ into each expression. Check: $2\times 19+2\times 11=60$. The area is 209 m².
d. $60\times 14.50=$ **£870**. Multiply the perimeter by the price per metre. The fencing costs £870.

Answer to Question 9

a. City Cabs: $2.40+1.60d$. Quick Cars: $3.80+1.40d$.
b. $2.40+1.60d=3.80+1.40d$, so $0.2d=1.4$ and $d=$ **7 miles**. The cost is $2.40+11.20=$ **£13.60**.
c. 3 miles: City Cabs $=£7.20$, Quick Cars $=£8.00$. **City Cabs** is cheaper by **80p**.
d. 12 miles: City Cabs $=£21.60$, Quick Cars $=£20.60$. **Quick Cars** is cheaper by **£1.00**.

Answer to Question 10

a. Child tickets $=120-a$. Every ticket is either adult or child, so the child tickets are the remaining $120-a$.
b. $25a+15(120-a)=2400$, so $25a+1800-15a=2400$, so $10a+1800=2400$. Multiply each number of tickets by its price and add. Expand the bracket and collect the a terms.
c. $10a=600$, so $a=60$ adult tickets and **60** child tickets. Subtract 1800 from both sides, then divide by 10. Check: $25\times 60+15\times 60=1500+900=2400$.
d. $10a+1800\geq 2700$, so $a\geq 90$: at least **90 adult tickets**. Write the takings as an inequality and solve it like an equation. The school must sell at least 90 adult tickets.

Answer to Question 11

a. $48n\geq 212+8=220$. Everyone needs a seat: 212 pupils plus 8 teachers make 220 people, and n coaches seat $48n$ people.
b. $n\geq \dfrac{220}{48}=4.58$, so the school needs **5 coaches**. Divide both sides by 48. We round up because 4 coaches seat only 192 people. The school needs 5 coaches.
c. $5\times 350=$ **£1750**. Multiply the number of coaches by the cost of one coach. The coaches cost £1750.
d. $350n\leq 1500$, so $n\leq 4.29$, so at most **4 coaches**, carrying $4\times 48=192$ people,

which is not enough for everyone. We round down because 5 coaches would cost more than £1500.

Answer to Question 12

a. Savings $= 35 + 8w$. She starts with £35 and adds £8 each week, so after w weeks she has $35 + 8w$ pounds.
b. $35 + 8w \geq 150$. She can afford the console when her savings are at least £150, so we use $\geq$.
c. $8w \geq 115$, $w \geq 14.375$, so **15 weeks**. Subtract 35 from both sides, then divide by 8. We round up because after 14 weeks she has only £147. She needs 15 weeks.
d. With £55: $55 + 8w \geq 150$, $8w \geq 95$, $w \geq 11.875$, so **12 weeks**. She now starts with $35 + 20 = 55$ pounds. After 11 weeks she has £143, which is not enough, so she needs 12 weeks.

Answer to Question 13

a. Fit4U: $30 + 18m$. Gymzone: $25m$. Fit4U adds £18 each month to the £30 fee; Gymzone charges £25 each month.
b. $30 < 7m$, so $m > 4.29$. Subtract $18m$ from both sides, then divide by 7: $m > \frac{30}{7}$.
c. **5 months** or more (after 5 months: Fit4U £120, Gymzone £125). The number of months must be a whole number greater than 4.29, so Fit4U is cheaper from 5 months onwards.
d. 12 months: Fit4U $= 30 + 216 = £246$, Gymzone $= £300$. She saves **£54**. Twelve months is more than 5 months, so Fit4U is cheaper and Alisha saves £54.

Answer to Question 14

a. $(x + 30) + 2(2x - 5) = 180$, so $x + 30 + 4x - 10 = 180$, so $5x + 20 = 180$. An isosceles triangle has two equal base angles, so we use $2(2x - 5)$ for both of them.
b. $5x = 160$, so $x = 32$. Subtract 20 from both sides, then divide by 5.
c. Apex $= 32 + 30 =$ **62°**. Each base angle $= 2 \times 32 - 5 =$ **59°**. Substitute $x = 32$ into each expression.
d. $62 + 59 + 59 = 180$ ✓ The angles add up to $180°$, so the answer is correct.

Answer to Question 15

a. $4(x + 3) = 2(2x - 1 + x + 2)$, so $4x + 12 = 2(3x + 1) = 6x + 2$. The square has four equal sides; the rectangle has two lengths and two widths.
b. $10 = 2x$, so $x = 5$. Subtract $4x$ and 2 from both sides, then divide by 2.
c. Square side $=$ **8 cm**. Rectangle $= 9$ cm by 7 cm (perimeter $2(9 + 7) = 32 = 4 \times 8$

✓).

d. Square area $= 64$ cm². Rectangle area $= 63$ cm². The **square is bigger by 1 cm²**. Equal perimeters do not mean equal areas.

Answer to Question 16

a. Profit $= 3d - 1.2d - 60 = 1.8d - 60$. Each drink makes $3 - 1.20 = 1.80$ pounds before costs, and the daily costs of £60 are subtracted.

b. $1.8d - 60 > 0$, $d > 33.3$, so at least **34 drinks**. A profit means the expression is greater than 0. We round up because 33 drinks give a small loss. The café needs at least 34 drinks.

c. $1.8d - 60 \geq 100$, $1.8d \geq 160$, $d \geq 88.9$, so at least **89 drinks**. Add 60 to both sides, then divide by 1.8. We round up because 88 drinks give only £98.40.

d. $1.8 \times 80 - 60 = 144 - 60 =$ **£84**. The target of £100 is **not** reached. So 80 drinks give a profit of £84, which is £16 short of the target.

Answer to Question 17

a. $(x-2)(x-3) < 0$. The graph is below the axis between the roots: $2 < x < 3$. The graph of $y = (x-2)(x-3)$ is a U-shape crossing the axis at 2 and 3, and it is below the axis between them. The solution is $2 < x < 3$.

b. $x^2 \geq 16$ gives $x \leq -4$ **or** $x \geq 4$. The roots of $x^2 = 16$ are -4 and 4. The squares are at least 16 outside these values, so $x \leq -4$ or $x \geq 4$.

c. $(x-4)(x+2) > 0$. The graph is above the axis outside the roots: $x < -2$ **or** $x > 4$. Factorise to find the roots -2 and 4. We want the parts of the U-shaped graph above the axis.

d. $-3.16 < x < 3.16$, so the integers are **−3, −2, −1, 0, 1, 2, 3** (7 integers). The roots of $x^2 = 10$ are $\pm\sqrt{10} = \pm 3.16$, so $-3.16 < x < 3.16$ and there are 7 integers.

Answer to Question 18

a. $x = 1$ (solid), $y = 5$ (solid) and $y = x$ (dashed, because the inequality is $>$ not $\geq$).

b. $y > 1$ and $y \leq 5$: $(1,2), (1,3), (1,4), (1,5)$.

c. $x = 1$: 4 points. $x = 2$: $(2,3), (2,4), (2,5)$. $x = 3$: $(3,4), (3,5)$. $x = 4$: $(4,5)$.

d. There are $4 + 3 + 2 + 1 =$ **10 points**. $(3,3)$ is **not** in R because $y > x$ needs $3 > 3$, which is false (it lies on the dashed line).

Answer to Question 19

a. Multiply by 6: $2(2x+1) = 3(x-4)$, $4x + 2 = 3x - 12$, so $x = -14$. The common denominator of 3 and 2 is 6. Check: $\frac{-27}{3} = -9$ and $\frac{-18}{2} = -9$.

b. Multiply by 15: $5x + 3(x - 2) = 30$, $8x - 6 = 30$, $8x = 36$, so $x = 4.5$. The common denominator of 3 and 5 is 15. Check: $1.5 + 0.5 = 2$.

c. $\frac{n + 7}{3} = n - 5$, $n + 7 = 3n - 15$, $22 = 2n$, so $n = 11$. Let the number be n and write the sentence as an equation, then multiply both sides by 3.

d. $\frac{11 + 7}{3} = \frac{18}{3} = 6$ and $11 - 5 = 6$ ✓ Both sides give 6, so the number is 11.

Answer to Question 20

a. $(n + 2) + (n + 4) = n + 22$, so $2n + 6 = n + 22$, $n = 16$: the numbers are **16, 18, 20**.

b. $n + (n + 2) + (n + 4) = 3n + 6 = 3(n + 2)$, and since $n + 2$ is even, $3(n + 2) = 6 \times \frac{n + 2}{2}$, a multiple of 6.

c. $x = \frac{12 - k}{3} > 0$, so $k < 12$. Make x the subject, then the solution is positive when $12 - k > 0$, which gives $k < 12$.

d. For $x = 2$ (the smallest positive even number): $k = 12 - 6 = 6$. For $x = 4$: $k = 0$; for larger x, k is smaller. So the largest is $k = 6$.