Lesson - Indices and standard form

Year: Year 10

Indices (powers) and standard form let us write very large and very small numbers compactly and simplify algebra quickly. On the GCSE you need the index laws with integer powers, negative and zero powers, standard form calculations with and without a calculator, and (Higher tier) fractional indices and estimates of powers and roots.

Starter activity

Light takes about 8 minutes to reach us from the Sun, 1.3 seconds from the Moon and 4 years from the nearest star. Write each of these times in seconds and try to put them in order on a number line. Which numbers are easy to compare and which are hard? Now write the number of seconds in a year (about 31 500 000) in a shorter way. What would you need to write the age of the universe, about 13 800 000 000 years, in seconds?

Powers and roots

Definition: power (index) and root

a^n means a multiplied by itself n times: $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. Learn the squares up to 15^2, the cubes of 1 to 5 and 10, and powers of 2 up to $2^{10} = 1024$. Roots reverse powers: $\sqrt{49} = 7$ and $\sqrt[3]{64} = 4$.

Key fact: numbers to learn

Know the squares up to $15^2 = 225$, the cubes of 1 to 5 and 10, and the powers of 2 up to $2^{10} = 1024$.

The index laws

Rule: the index laws

Law	Example
$a^m \times a^n = a^{m+n}$	$a^3 \times a^5 = a^8$
$a^m \div a^n = a^{m-n}$	$7^9 \div 7^6 = 7^3$
$(a^m)^n = a^{mn}$	$(2^3)^4 = 2^{12}$
$a^0 = 1$	$5^0 = 1$
$a^{-n} = \frac{1}{a^n}$	$2^{-3} = \frac{1}{8}$

The laws apply only when the base is the same. For terms with coefficients, deal with the numbers and each letter separately: $3x^2 \times 4x^5 = 12x^7$ and $(2x^3)^4 = 2^4x^{12} = 16x^{12}$.

A negative power does *not* make the answer negative: it means "reciprocal". $4^{-2} = \frac{1}{16}$.

Fractional indices (Higher tier)

Rule: fractional indices (Higher tier)

$a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$. Take the root first because it keeps numbers small: $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$. Combine with a negative power by taking the reciprocal: $16^{-\frac{3}{4}} = \frac{1}{16^{3/4}} = \frac{1}{2^3} = \frac{1}{8}$.

Standard form

Definition: standard form

A number is in standard form when it is written as $A \times 10^n$ with $1 \leq A$ and n a whole number. Large numbers have positive n; numbers less than 1 have negative n.

- $5\,600\,000 = 5.6 \times 10^6$ (move the decimal point 6 places left).
- $0.000072 = 7.2 \times 10^{-5}$ (move the decimal point 5 places right).
- Be careful: 12×10^4 is not in standard form. It is 1.2×10^5.

Calculating in standard form

Method: calculating in standard form

Multiply and divide: deal with the numbers and the powers of 10 separately. $(4 \times 10^3) \times (2.5 \times 10^5) = 10 \times 10^8 = 1 \times 10^9$. $(6 \times 10^8) \div (3 \times 10^3) = 2 \times 10^5$. Always check that the final answer is in standard form.

Add and subtract: write both with the same power of 10 first. $3.2 \times 10^5 + 4.5 \times 10^4 = 3.2 \times 10^5 + 0.45 \times 10^5 = 3.65 \times 10^5$.

On a calculator use the $\times 10^x$ key (or EXP). Do not type "$\times 10$" then the ^ key unless you use brackets.

Estimating (Higher tier)

Method: estimating with powers and roots (Higher tier)

To estimate a calculation, round each number to 1 significant figure. $\frac{\sqrt{99}\times 3.9}{0.51}\approx\frac{10\times 4}{0.5}=80$. For estimates of roots choose a nearby perfect square: $\sqrt{50}$ is a little more than 7 because $7^2=49$.

Worked examples

Worked example 1

Light travels at 3×10^5 km/s.

The Sun is 1.5×10^8 km away. How long does light take to reach us?
$\text{time}=\frac{1.5\times10^8}{3\times10^5}=0.5\times10^3=500$ s $=8$ min 20 s.

Worked example 2

Simplify $\frac{12a^7}{3a^2}$.

Divide the numbers: $12\div3=4$. Subtract the powers: $a^{7-2}=a^5$. Answer $4a^5$.

Worked example 3 (Higher tier)

Solve $3^{2x-1}=81$.

$81=3^4$, so $2x-1=4$, $x=\frac{5}{2}$.

More worked examples in exam style

Worked example 4

Simplify $\frac{(3a^2b)^3}{9ab^2}$ [3 marks].

Cube the bracket: $27a^6b^3$. Divide: $\frac{27}{9}=3$, $a^{6-1}=a^5$, $b^{3-2}=b$. Answer $3a^5b$.

Worked example 5

The population of a country is 6.7×10^7 and its area is 2.4×10^5 km^2. Find the population per km^2 to 3 significant figures [3 marks].

$\frac{6.7\times10^7}{2.4\times10^5}=2.79\ldots\times10^2=279$ people per km^2.

Worked example 6

Write $\dfrac{2^8 \times 2^{-3}}{2^2}$ as a single power of 2 [2 marks].

Top: $2^{8-3} = 2^5$. Then $2^{5-2} = 2^3$.

Worked example 7 (Higher tier)

Work out $\left(\dfrac{8}{27}\right)^{-\frac{2}{3}}$ [3 marks].

Reciprocal: $\left(\frac{27}{8}\right)^{\frac{2}{3}}$. Cube root: $\frac{3}{2}$. Square: $\frac{9}{4}$.

Common mistakes

Watch out

- Multiplying the indices when you should add them ($a^3 \times a^5$ is a^8, not a^{15}).
- Writing 2^{-3} as -8 or -6.
- Forgetting to square the coefficient in $(2x^3)^4$.
- Leaving a standard form answer as 25×10^6, which is not standard form.
- Adding the powers of 10 when adding standard form numbers.

Exam technique

Exam technique

- Write each step of an index simplification on its own line so that a slip in one law does not lose all the marks.
- In standard form questions with a calculator, write down the unrounded value as well as the final answer.
- Check the sign of the power of 10: a number smaller than 1 always has a negative power.
- For fractional indices always take the root first, then the power.

Why standard form is useful

Scientists, engineers and economists often work with numbers that are too big or too small to write in full, such as the distance to the Sun, the size of an atom or the national debt. Standard form writes a number as $a \times 10^n$ where a is at least 1 and less than 10 and n is a whole number, positive for large numbers and negative for small ones. It makes comparison fast, because the power of 10 tells you the size of the number, and it makes multiplication and division simple, because you can multiply the a parts and add the powers of 10. The index laws are exactly what makes this work: $10^m \times 10^n = 10^{m+n}$ and

$10^m \div 10^n = 10^{m-n}$. For GCSE you also need to enter standard form on a calculator using the $\times 10^x$ key, and to read calculator displays such as 4.5E7 as 4.5×10^7.

Worked example 8: adding numbers in standard form

Work out $(6.4 \times 10^5) + (2.5 \times 10^4)$ and give the answer in standard form [3 marks].

You cannot add the a parts until the powers of 10 match. Write $2.5 \times 10^4 = 0.25 \times 10^5$. Then $6.4 \times 10^5 + 0.25 \times 10^5 = 6.65 \times 10^5$. Check with ordinary numbers: $640\,000 + 25\,000 = 665\,000$. This is 6.65×10^5, so the answer is correct.

Worked example 9: a calculation that needs adjusting

Work out $(5 \times 10^7) \times (4 \times 10^{-3})$ in standard form [2 marks].

Multiply the numbers: $5 \times 4 = 20$. Add the powers: $10^{7+(-3)} = 10^4$. The product is 20×10^4, which is not standard form because 20 is not less than 10. Adjust: $20 = 2 \times 10$, so $20 \times 10^4 = 2 \times 10^5$. The final answer is 2×10^5.

Worked example 10: simplifying with negative indices

Simplify $\dfrac{x^{-2} \times x^5}{x^3}$ and write the answer without negative indices [3 marks].

Multiply on top: $x^{-2} \times x^5 = x^{-2+5} = x^3$. Divide: $\dfrac{x^3}{x^3} = x^{3-3} = x^0 = 1$. A quick check with $x = 2$: $\dfrac{2^{-2} \times 2^5}{2^3} = \dfrac{\frac{1}{4} \times 32}{8} = \dfrac{8}{8} = 1$. The answer is 1.

Worked example 11: roots and powers together

Work out $\sqrt{2.25 \times 10^6}$ without a calculator [2 marks].

Split the number: $\sqrt{2.25} \times \sqrt{10^6}$. We know $1.5^2 = 2.25$ and $\sqrt{10^6} = 10^3$. So the answer is $1.5 \times 10^3 = 1500$. Taking the square root of a power of 10 halves the index, which is why the power was written as an even number.

Note: using the calculator

On most calculators the standard form button is labelled **EXP** or **×10ˣ**. To enter 3.2×10^{-5} press 3.2, then the button, then (−) and 5. Do *not* type $\times 10$ and then the power, because the calculator will treat it as a normal calculation and the order of operations may change your answer. Always write the calculator answer in full standard form in the exam.

Watch out: negative indices

A negative index does not make the number negative: $2^{-3} = \frac{1}{8}$, not -8. It means "divide" rather than "multiply". Also remember that $(-2)^2 = 4$ but $-2^2 = -4$.

Putting the laws to work

Exam questions on indices rarely test one law on its own. A typical question mixes numbers and letters, negative and fractional powers, and asks for an answer in a particular form, such as a single power of 2 or a fraction without negative indices. The best approach is to deal with the numbers first, then each letter in turn, writing one law at a time. When the bases are different numbers, try to write them as powers of the same prime. For instance $4^3 \times 8^2$ becomes $(2^2)^3 \times (2^3)^2 = 2^6 \times 2^6 = 2^{12}$, which is much easier to compare with other powers of 2. When you finish, always ask yourself whether the answer is in the form the question asked for.

Worked example 12: different bases

Write $\frac{9^3 \times 27}{81^2}$ as a power of 3 and find its value [3 marks].

Write every number as a power of 3: $9^3 = (3^2)^3 = 3^6$, $27 = 3^3$ and $81^2 = (3^4)^2 = 3^8$. The top is $3^6 \times 3^3 = 3^9$. Divide: $\frac{3^9}{3^8} = 3^{9-8} = 3^1 = 3$. So the answer is 3. Check with a calculator: $729 \times 27 = 19\,683$ and $81^2 = 6561$, and $19\,683 \div 6561 = 3$.

Did you know?

Large numbers have been named for a long time: million, billion and trillion, but the names are used differently in different places. In the UK a billion used to mean a million million, but today it means a thousand million, as in the United States. The physicist Richard Feynman once said that a million is hard to imagine, and that if you counted one number every second it would take about 12 days to count to a million and about 32 years to count to a billion. The Indian mathematician Aryabhata and later Indian scholars developed place value around 500 AD, and Archimedes wrote a book called *The Sand Reckoner* in which he invented a way to name very large numbers in order to count the grains of sand that could fill the universe.

Otto's tip

Break every index question into three jobs: the number, then each letter, then the powers of 10. Finish by asking: is my answer in the form A times 10 to the power n with A between 1 and 10?