# Worksheet - Indices and standard form

Year: Year 10 · 20

Show your working.

## Question 1

**Question 1 – Index laws with numbers and letters** *(Section A · Skills practice)*

Non-calculator. Simplify each expression, leaving numbers as powers where sensible.

a. $3^5 \times 3^4$ and $7^8 \div 7^3$
b. $(2^3)^4$ and $5^0$
c. $2x^3 \times 4x^5$
d. $\dfrac{12a^7}{3a^2}$ and $(3b^2)^3$

## Question 2

**Question 2 – Negative and zero indices** *(Section A · Skills practice)*

Non-calculator. Work out the value of each as an integer or a fraction.

a. $5^0$ and $10^{-3}$
b. $4^{-2}$
c. $\left(\dfrac{1}{3}\right)^{-2}$
d. $2^{-1} + 2^{-2}$

## Question 3

**Question 3 – Powers and roots** *(Section A · Skills practice)*

Non-calculator. Write down the value of each.

a. $\sqrt{144}$ and $\sqrt[3]{125}$
b. $2^6$ and $15^2$
c. $\sqrt[3]{-27}$ and $(-4)^3$
d. $\sqrt{0.09}$ and $\sqrt{\dfrac{49}{100}}$

## Question 4

**Question 4 – Writing numbers in standard form** *(Section A · Skills practice)*

Write each number in standard form $a \times 10^n$ where $1 \leq a < 10$. Then write the last two as ordinary numbers.

a. $4\,500\,000$ and $83\,000\,000\,000$
b. $0.00072$ and $0.0000605$
c. $3.2 \times 10^4$
d. $7.05 \times 10^{-3}$

## Question 5

**Question 5 – Calculating in standard form** *(Section A · Skills practice)*

Non-calculator. Give each answer in standard form.

a. $(3 \times 10^4) \times (2 \times 10^3)$
b. $(8 \times 10^6) \div (2 \times 10^2)$
c. $(4.2 \times 10^5) + (3 \times 10^4)$
d. $(6.3 \times 10^3) \times (5 \times 10^{-5})$

## Question 6

**Question 6 – Ordering numbers in standard form** *(Section A · Skills practice)*

Put these numbers in order, smallest first. Compare the powers of 10 first.

a. $3.2 \times 10^5$, $4.1 \times 10^4$, $9.8 \times 10^5$, $2.5 \times 10^{-1}$
b. $6 \times 10^{-3}$, $5.9 \times 10^{-3}$, $0.0061$, $6.01 \times 10^{-4}$
c. Which is bigger, $7.3 \times 10^8$ or $2.1 \times 10^9$? Explain.
d. Write $2^{10}$, $10^3$ and $3^7$ in order of size, smallest first.

## Question 7

**Question 7 – Light from the sun** *(Section B · Real-life practice)*

The Sun is $1.5 \times 10^8$ km from the Earth. Light travels at $3 \times 10^5$ km per second. Calculator allowed.

a. How long does light take to reach the Earth from the Sun, in seconds?
b. Write your answer to part (a) in minutes and seconds.
c. The Moon is $3.84 \times 10^5$ km from the Earth. How long does light take from the Moon, to 3 significant figures?
d. How many times further is the Sun than the Moon? Give your answer to the nearest whole number.

## Question 8

**Question 8 – How many people?** *(Section B · Real-life practice)*

The population of the world is about $8.1 \times 10^9$ and the population of the UK is about $6.7 \times 10^7$. Calculator allowed.

a. How many times bigger is the world population than the UK population? Give your answer to the nearest whole number.
b. What percentage of the world population lives in the UK, to 2 decimal places?
c. The UK's land area is $2.4 \times 10^5$ km². Find the population density in people per km² to the nearest whole number.
d. By 2050 the UK population is expected to grow by 10%. Write the new population in standard form.

## Question 9

**Question 9 – The size of atoms** *(Section B · Real-life practice)*

A hydrogen atom has a diameter of about $1.06 \times 10^{-10}$ metres. Calculator allowed.

a. How many hydrogen atoms in a line would fit across a distance of 1 mm ($10^{-3}$ m)? Give your answer in standard form to 3 s.f.
b. A grain of sand has a diameter of $2 \times 10^{-4}$ m. How many atoms would fit across the grain?
c. A human hair has a width of $7 \times 10^{-5}$ m. How many times wider is the hair than one atom, to 2 s.f.?
d. Write the diameter of the atom in nanometres, using $1\ \text{nm} = 10^{-9}$ m.

## Question 10

**Question 10 – Storing photos** *(Section B · Real-life practice)*

One terabyte (1 TB) is $10^{12}$ bytes. A photo needs about $4 \times 10^6$ bytes and a song needs about $5 \times 10^6$ bytes. Calculator allowed.

a. How many photos fit on a 1 TB drive?
b. How many songs fit on a 1 TB drive?
c. A drive holds 2000 songs and 50 000 photos. How many bytes is that, in standard form?
d. What fraction of 1 TB is used in part (c)? Give your answer as a percentage.

## Question 11

**Question 11 – Lines of cells and bacteria** *(Section B · Real-life practice)*

A type of bacterium is $2.5 \times 10^{-6}$ m long. Calculator allowed.

a. How many of these bacteria placed end to end would stretch 1 metre?
b. They are placed in a line along the length of a 15 cm ruler. How many bacteria is

that?

c. The bacteria double in number every 20 minutes. Starting with 500, how many are there after 6 hours? Give your answer in standard form to 3 s.f.
d. How many hours until there are more than $10^{12}$ bacteria?

## Question 12

**Question 12 – Folding paper** *(Section B · Real-life practice)*

A sheet of paper is 0.1 mm thick. Each fold doubles the thickness.

| Folds | Thickness (mm) |
|---|---|
| 0 | 0.1 |
| 1 | 0.2 |
| 2 | 0.4 |
| 3 | 0.8 |

a. Write a formula for the thickness $T$ mm after $n$ folds.
b. Find the thickness after 10 folds.
c. Find the thickness after 20 folds in metres, to 3 significant figures, in standard form.
d. How many folds are needed to get thicker than the height of a 2 m door?

## Question 13

**Question 13 – Money and the national debt** *(Section B · Real-life practice)*

At one time the UK national debt was about £$2.7 \times 10^{12}$. The UK population was about $6.7 \times 10^{7}$. Calculator allowed.

a. Work out the debt per person, in standard form to 2 s.f.
b. A family of four has a yearly income of £$6.4 \times 10^{4}$. How many years of their income equals their share of the debt?
c. Each year the debt grows by 2.5%. By how many pounds does it grow in the first year? Give your answer in standard form.
d. What is the new debt after the first year, in standard form to 3 s.f.?

## Question 14

**Question 14 – Power and energy** *(Section B · Real-life practice)*

A wind farm produces $1.8 \times 10^{7}$ watts. A kettle uses $2.4 \times 10^{3}$ watts. Calculator allowed.

a. How many kettles could the wind farm run at the same time?
b. The farm runs for $8 \times 10^{3}$ hours in a year. How many watt-hours does it produce?

Give your answer in standard form.

c. A house uses $3.2 \times 10^3$ kilowatt-hours per year. If 1 kilowatt-hour is $10^3$ watt-hours, how many houses could be supplied by the farm's yearly output?

d. Write the yearly output in kilowatt-hours.

## Question 15

**Question 15 – Higher: fractional indices** *(Section C · Problem solving)*

Higher tier. Use $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$. Non-calculator.

a. $25^{\frac{1}{2}}$ and $27^{\frac{1}{3}}$

b. $8^{\frac{2}{3}}$ and $16^{\frac{3}{4}}$

c. $27^{-\frac{2}{3}}$ and $\left(\frac{1}{4}\right)^{-\frac{3}{2}}$

d. Solve $2^x = \frac{1}{32}$ and $9^x = 27$.

## Question 16

**Question 16 – Proving with indices** *(Section C · Problem solving)*

Use the index laws to prove each statement for any whole number $n$.

a. Show that $2^n + 2^{n+1} = 3 \times 2^n$.

b. Show that $5^{n+2} - 5^n = 24 \times 5^n$.

c. Show that $4^{n+1}$ is a multiple of 4.

d. Show that $\frac{6^{n+1}}{2^n} = 6 \times 3^n$.

## Question 17

**Question 17 – Solving equations with indices** *(Section C · Problem solving)*

Write both sides as powers of the same base, then compare the indices.

a. $3^{x+1} = 81$

b. $2^{2x} = 64$

c. $5^x = \frac{1}{25}$

d. Higher tier: $4^x = 8$.

## Question 18

**Question 18 – Higher: estimating** *(Section C · Problem solving)*

Higher tier. Round each number to 1 significant figure to estimate. Then use a calculator and find the percentage error of your estimate to 1 d.p.

a. Estimate $\dfrac{4.8 \times 19.7}{0.21}$.
b. Estimate $\sqrt{\dfrac{9.2 \times 51}{1.9}}$.
c. Work out the accurate value of (a) to 2 d.p. and the percentage error of your estimate.
d. Explain whether your estimate for (a) is an overestimate or underestimate, and why.

## Question 19

**Question 19 – Planets in standard form** *(Section C · Problem solving)*

The mass of the Sun is $1.99 \times 10^{30}$ kg and the mass of the Earth is $5.97 \times 10^{24}$ kg. Jupiter's mass is $1.9 \times 10^{27}$ kg.

a. How many times heavier is the Sun than the Earth, in standard form to 3 s.f.?
b. How many times heavier is Jupiter than the Earth, to 3 s.f.?
c. What fraction of the Sun's mass is Jupiter, in standard form to 2 s.f.?
d. Find the combined mass of the Earth and Jupiter in standard form to 3 s.f.

## Question 20

**Question 20 – Challenge: big powers** *(Section D · Challenge)*

Calculator allowed.

a. Which is bigger, $2^{100}$ or $10^{30}$? Use $2^{10} = 1024$ to explain.
b. How many digits does $2^{100}$ have?
c. The last digits of powers of 7 repeat: 7, 9, 3, 1, ... What is the last digit of $7^{2025}$?
d. Show that $2^{10} \approx 10^3$ and use it to estimate $2^{40}$ in standard form.