Answers - Indices and standard form

Year: Year 10

Answer to Question 1

a. $3^{5+4} = 3^9$ and $7^{8-3} = 7^5$ When we multiply powers of the same base we add the indices, and when we divide we subtract them.

b. $2^{3\times4} = 2^{12}$ and $5^0 = \mathbf{1}$ A power of a power means we multiply the indices. Any non-zero number to the power 0 is 1.

c. $2 \times 4 = 8$ and $x^3 \times x^5 = x^8$, so $8x^8$ Multiply the numbers and add the indices of x separately. The answer is $8x^8$.

d. $12 \div 3 = 4$ and $a^{7-2} = a^5$, so $4a^5$. $(3b^2)^3 = 3^3b^6 = 27b^6$ Divide the numbers and subtract the indices. For $(3b^2)^3$, cube the 3 and multiply the index 2 by 3.

Answer to Question 2

a. $5^0 = \mathbf{1}$ and $10^{-3} = \dfrac{1}{1000} = \mathbf{0.001}$ Any non-zero number to the power 0 equals 1. A negative index means the reciprocal, so $10^{-3} = \frac{1}{10^3}$.

b. $4^{-2} = \dfrac{1}{4^2} = \dfrac{1}{16}$ The negative index gives the reciprocal of $4^2 = 16$. So $4^{-2} = \frac{1}{16}$.

c. $\left(\dfrac{1}{3}\right)^{-2} = 3^2 = \mathbf{9}$ Turn the fraction upside down to make the index positive, then square. So the value is 9.

d. $\dfrac{1}{2} + \dfrac{1}{4} = \dfrac{3}{4}$ Write each power as a fraction, then use the common denominator 4: $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$.

Answer to Question 3

a. **12** and **5** $12 \times 12 = 144$ and $5 \times 5 \times 5 = 125$, so the square root of 144 is 12 and the cube root of 125 is 5.

b. **64** and **225** $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$ and $15^2 = 15 \times 15 = 225$.

c. **−3** and **−64** A negative number has a negative cube root because $(-3)^3 = -27$. An odd power of a negative number is negative, so $(-4)^3 = -64$.

d. **0.3** and $\dfrac{7}{10}$ **(0.7)** $0.3 \times 0.3 = 0.09$, and the square root of a fraction is the square root of the top over the square root of the bottom: $\frac{7}{10}$.

Answer to Question 4

a. 4.5×10^{6} and 8.3×10^{10} Put the decimal point after the first non-zero digit, then count how many places it moves. The point moves 6 and 10 places to the left, so the powers are positive.
b. 7.2×10^{-4} and 6.05×10^{-5} For small numbers the point moves to the right, so the powers are negative: 4 places and 5 places.
c. **32 000** Multiply 3.2 by $10^{4} = 10\,000$, which moves the point 4 places to the right. So $3.2 \times 10^{4} = 32\,000$.
d. **0.00705** A power of -3 means divide by 1000, which moves the point 3 places to the left. So $7.05 \times 10^{-3} = 0.00705$.

Answer to Question 5

a. $3 \times 2 = 6$ and $10^{4+3} = 10^{7}$: 6×10^{7} Multiply the numbers, then add the powers of 10. The answer is 6×10^{7}.
b. $8 \div 2 = 4$ and $10^{6-2} = 10^{4}$: 4×10^{4} Divide the numbers, then subtract the powers of 10. The answer is 4×10^{4}.
c. $420\,000 + 30\,000 = 450\,000 = 4.5 \times 10^{5}$ To add numbers in standard form, change them to ordinary numbers (or to the same power of 10) first. The answer is 4.5×10^{5}.
d. $6.3 \times 5 = 31.5$ and $10^{3-5} = 10^{-2}$, giving $31.5 \times 10^{-2} = 3.15 \times 10^{-1}$ 31.5 is not between 1 and 10, so we write $31.5 = 3.15 \times 10^{1}$ and add the powers: $10^{1} \times 10^{-2} = 10^{-1}$.

Answer to Question 6

a. 2.5×10^{-1}, 4.1×10^{4}, 3.2×10^{5}, 9.8×10^{5} The power of 10 decides the size first; only when the powers are equal do we compare the numbers in front.
b. 6.01×10^{-4}, 5.9×10^{-3}, 6×10^{-3}, 0.0061 ($0.0061 = 6.1 \times 10^{-3}$)
c. $2.1 \times 10^{9} = 21 \times 10^{8}$, which is bigger than 7.3×10^{8}, so 2.1×10^{9} is bigger (higher power of 10).
d. $2^{10} = 1024$, $10^{3} = 1000$, $3^{7} = 2187$. In order: 10^{3}, 2^{10}, 3^{7}.

Answer to Question 7

a. Time $= \dfrac{1.5 \times 10^{8}}{3 \times 10^{5}} = 0.5 \times 10^{3} =$ **500 s** (5×10^{2}). Time = distance ÷ speed.
Divide the numbers and subtract the powers of 10. Light takes 500 seconds to reach the Earth.

b. $500 \div 60 = 8$ remainder 20, so **8 minutes 20 seconds**. 8 minutes is 480 seconds, which leaves 20 seconds. So light takes 8 minutes 20 seconds.
c. $\frac{3.84 \times 10^5}{3 \times 10^5} =$ **1.28 s**. The powers of 10 are equal, so they cancel. Light takes 1.28 seconds to reach us from the Moon.
d. $\frac{1.5 \times 10^8}{3.84 \times 10^5} = 390.6$, so about **391 times** further. Divide the larger distance by the smaller one. The Sun is about 391 times further away than the Moon.

Answer to Question 8

a. $\frac{8.1 \times 10^9}{6.7 \times 10^7} = 120.9$, so about **121 times**. Divide the world population by the UK population. The world population is about 121 times bigger.
b. $\frac{6.7 \times 10^7}{8.1 \times 10^9} \times 100 =$ **0.83%**. Divide the UK population by the world population and multiply by 100. About 0.83% of the world population lives in the UK.
c. $\frac{6.7 \times 10^7}{2.4 \times 10^5} = 279.2$, so about **279 people per km²**. Population density is the number of people divided by the area. There are about 279 people per km².
d. $6.7 \times 10^7 \times 1.1 = 7.37 \times 10^7$, so 7.37×10^7. A 10% increase gives the multiplier 1.1. The new population is 7.37×10^7.

Answer to Question 9

a. $\frac{10^{-3}}{1.06 \times 10^{-10}} = 0.943 \times 10^7 = 9.43 \times 10^6$ Divide the distance by the diameter of one atom. 0.943 is not between 1 and 10, so we write it as 9.43×10^{-1} and the answer becomes 9.43×10^6 atoms.
b. $\frac{2 \times 10^{-4}}{1.06 \times 10^{-10}} = 1.887 \times 10^6 = 1.89 \times 10^6$ atoms. Divide the diameter of the grain by the diameter of an atom. About 1.89×10^6 atoms fit across the grain.
c. $\frac{7 \times 10^{-5}}{1.06 \times 10^{-10}} = 6.6 \times 10^5 = 6.6 \times 10^5$ times. Divide the width of the hair by the diameter of an atom. The hair is about 6.6×10^5 times wider.
d. 1.06×10^{-10} m $= 0.106 \times 10^{-9}$ m $=$ **0.106 nm**. One nanometre is 10^{-9} m, so we divide by 10^{-9}. The atom is 0.106 nm across.

Answer to Question 10

a. $\frac{10^{12}}{4 \times 10^{6}} = 0.25 \times 10^{6} = 2.5 \times 10^{5}$ **(250 000) photos**. Divide the size of the drive by the size of one photo. About 250 000 photos fit on the drive.

b. $\frac{10^{12}}{5 \times 10^{6}} = 2 \times 10^{5}$ **(200 000) songs**. Divide the size of the drive by the size of one song. About 200 000 songs fit on the drive.

c. Songs: $2000 \times 5 \times 10^{6} = 10^{10}$. Photos: $50\,000 \times 4 \times 10^{6} = 2 \times 10^{11}$. Total $= 2.1 \times 10^{11}$ bytes. Work out the songs and the photos separately, then add: $1 \times 10^{10} + 20 \times 10^{10} = 21 \times 10^{10}$. The total is 2.1×10^{11} bytes.

d. $\frac{2.1 \times 10^{11}}{10^{12}} = 0.21 =$ **21%**. Divide the bytes used by the size of the drive. So 21% of the drive is used.

Answer to Question 11

a. $\frac{1}{2.5 \times 10^{-6}} = 4 \times 10^{5}$.

b. 15 cm $= 0.15$ m: $0.15 \times 4 \times 10^{5} = 6 \times 10^{4}$. Change 15 cm into metres, then multiply by the number of bacteria in 1 m.

c. 6 hours is 18 doubling periods: $500 \times 2^{18} = 131\,072\,000 = 1.31 \times 10^{8}$.

d. Need $500 \times 2^{n} > 10^{12}$, so $2^{n} > 2 \times 10^{9}$. $2^{31} = 2.147 \times 10^{9}$ works and $2^{30} = 1.07 \times 10^{9}$ does not. So $n = 31$ periods $= 31 \times 20 = 620$ min $=$ **10 h 20 min**.

Answer to Question 12

a. $T = 0.1 \times 2^{n}$. The thickness starts at 0.1 mm and is multiplied by 2 for each fold, so after n folds it has been multiplied by 2^{n}.

b. $0.1 \times 2^{10} = 0.1 \times 1024 =$ **102.4 mm**. Substitute $n = 10$ into the formula. The thickness is 102.4 mm.

c. $0.1 \times 2^{20} = 104\,857.6$ mm $= 104.8576$ m $= 1.05 \times 10^{2}$ **m**. Divide by 1000 to change millimetres into metres. The paper would be about 1.05×10^{2} m thick.

d. Need $0.1 \times 2^{n} > 2000$ mm, so $2^{n} > 20\,000$. $2^{14} = 16\,384$ is too small and $2^{15} = 32\,768$ works. So **15 folds**. Change 2 m into 2000 mm so the units match. So 15 folds are needed.

Answer to Question 13

a. $\frac{2.7 \times 10^{12}}{6.7 \times 10^{7}} = 4.03 \times 10^{4} =$ **£**4.0×10^{4} per person. Divide the debt by the number of people. The debt is about £4.0×10^{4} per person.

b. Share for 4 people $= 1.612 \times 10^5$. $\dfrac{1.612 \times 10^5}{6.4 \times 10^4} =$ **2.5 years**. The family's share is 4 times the share per person. Divide it by the yearly income: their share equals about 2.5 years of income.
c. $2.7 \times 10^{12} \times 0.025 = 6.75 \times 10^{10}$, so **£**$6.75 \times 10^{10}$. 2.5% of the debt is $0.025 \times 2.7 \times 10^{12}$. The debt grows by £$6.75 \times 10^{10}$ in the first year.
d. $2.7 \times 10^{12} \times 1.025 = 2.7675 \times 10^{12} \approx$ **£**2.77×10^{12}. A 2.5% increase gives the multiplier 1.025. The new debt is about £2.77×10^{12}.

Answer to Question 14

a. $\dfrac{1.8 \times 10^7}{2.4 \times 10^3} = 0.75 \times 10^4 =$ **7500 kettles**. Divide the power of the farm by the power of one kettle. The farm could run 7500 kettles at once.
b. $1.8 \times 10^7 \times 8 \times 10^3 = 14.4 \times 10^{10} = 1.44 \times 10^{11}$ **Wh**. Energy in watt-hours is power times hours. Multiply the numbers and add the powers of 10, then rewrite 14.4 as 1.44×10^1.
c. A house uses $3.2 \times 10^3 \times 10^3 = 3.2 \times 10^6$ Wh. $\dfrac{1.44 \times 10^{11}}{3.2 \times 10^6} = 4.5 \times 10^4 =$ **45 000 houses**. Change the house's use into watt-hours so that the units match, then divide. About 45 000 houses could be supplied.
d. $1.44 \times 10^{11} \div 10^3 = 1.44 \times 10^8$ **kWh**. There are 1000 watt-hours in a kilowatt-hour, so we divide by 10^3. The output is 1.44×10^8 kWh.

Answer to Question 15

a. $\sqrt{25} =$ **5** and $\sqrt[3]{27} =$ **3** A power of $\frac{1}{2}$ is a square root and a power of $\frac{1}{3}$ is a cube root.
b. $(\sqrt[3]{8})^2 = 2^2 =$ **4** and $(\sqrt[4]{16})^3 = 2^3 =$ **8** The denominator of the index gives the root and the numerator gives the power. Take the root first to keep the numbers small.
c. $27^{-\frac{2}{3}} = \dfrac{1}{(\sqrt[3]{27})^2} = \dfrac{1}{9}$. $\left(\dfrac{1}{4}\right)^{-\frac{3}{2}} = 4^{\frac{3}{2}} = (\sqrt{4})^3 =$ **8**. So $\dfrac{1}{9}$ and **8**. A negative index means the reciprocal, so we turn the base upside down first.
d. $\dfrac{1}{32} = 2^{-5}$, so $x =$ **−5**. $9^x = 27$ means $3^{2x} = 3^3$, so $2x = 3$ and $x =$ **1.5**. Write both sides as powers of the same base, then set the indices equal.

Answer to Question 16

a. $2^{n+1} = 2 \times 2^n$, so $2^n + 2 \times 2^n = 3 \times 2^n$. ✓ Using $a^{m+n} = a^m \times a^n$, the term 2^{n+1} is twice 2^n. One lot of 2^n plus two lots makes three lots.
b. $5^{n+2} = 25 \times 5^n$, so $25 \times 5^n - 5^n = 24 \times 5^n$. ✓ Using the index laws, $5^{n+2} = 5^2 \times 5^n$. Twenty-five lots minus one lot leaves twenty-four lots of 5^n.
c. $4^{n+1} = 4 \times 4^n$, which is 4 times a whole number, so it is a **multiple of 4**. Since 4^n is a whole number for every whole number n, 4^{n+1} is a multiple of 4.
d. $\dfrac{6^{n+1}}{2^n} = \dfrac{6 \times 6^n}{2^n} = 6 \times \dfrac{6^n}{2^n} = 6 \times 3^n$. ✓ Powers with the same index can be divided base by base: $\frac{6^n}{2^n} = \left(\frac{6}{2}\right)^n = 3^n$.

Answer to Question 17

a. $81 = 3^4$, so $x + 1 = 4$ and $x =$ **3** Write 81 as a power of 3, then the indices must be equal. Check: $3^{3+1} = 3^4 = 81$.
b. $64 = 2^6$, so $2x = 6$ and $x =$ **3** Write 64 as a power of 2. Check: $2^{2\times3} = 2^6 = 64$.
c. $\dfrac{1}{25} = 5^{-2}$, so $x =$ **−2** Since $25 = 5^2$, its reciprocal is 5^{-2}. Check: $5^{-2} = \frac{1}{25}$.
d. $4 = 2^2$ and $8 = 2^3$, so $2^{2x} = 2^3$, $2x = 3$ and $x =$ **1.5** 4 and 8 are both powers of 2, so we write both sides with base 2. Check: $4^{1.5} = (\sqrt{4})^3 = 8$.

Answer to Question 18

a. $\dfrac{5 \times 20}{0.2} = \dfrac{100}{0.2} =$ **500**
b. $\sqrt{\dfrac{9 \times 50}{2}} = \sqrt{225} =$ **15**
c. Accurate value $\dfrac{4.8 \times 19.7}{0.21} = 450.29$. Error $= \dfrac{500 - 450.29}{450.29} \times 100 =$ **11.0%**.
d. It is an **overestimate**: 4.8 rounds up to 5, 19.7 up to 20, and 0.21 down to 0.2 in the denominator, so every rounding makes the answer bigger.

Answer to Question 19

a. $\dfrac{1.99 \times 10^{30}}{5.97 \times 10^{24}} = 0.3333 \times 10^6 = 3.33 \times 10^5$. Divide the numbers and subtract the powers of 10, then rewrite 0.3333 as 3.333×10^{-1}. The Sun is about 3.33×10^5 times heavier than the Earth.
b. $\dfrac{1.9 \times 10^{27}}{5.97 \times 10^{24}} = 0.3183 \times 10^3 =$ **318** times. Divide the masses in the same way. Jupiter is about 318 times heavier than the Earth.

c. $\frac{1.9 \times 10^{27}}{1.99 \times 10^{30}} = 0.955 \times 10^{-3} = 9.5 \times 10^{-4}$ (about 0.1%). Divide Jupiter's mass by the Sun's mass. Jupiter has about 9.5×10^{-4} of the Sun's mass.

d. $1.9 \times 10^{27} + 5.97 \times 10^{24} = 1.9 \times 10^{27} + 0.00597 \times 10^{27} = 1.90597 \times 10^{27} \approx 1.91 \times 10^{27}$ **kg**. To add, write both numbers with the same power of 10 first. The combined mass is about 1.91×10^{27} kg.

Answer to Question 20

a. $2^{100} = (2^{10})^{10} = 1024^{10}$, which is bigger than $1000^{10} = 10^{30}$. So 2^{100} **is bigger** (1.27×10^{30}).

b. $2^{100} = 1.27 \times 10^{30}$ has **31 digits**.

c. The pattern repeats every 4. $2025 \div 4 = 506$ remainder 1, so the last digit is the 1st in the cycle: **7**.

d. $1024 \approx 1000$ (2.4% bigger). $2^{40} = (2^{10})^4 \approx (10^3)^4 = 10^{12}$. The exact value is 1.0995×10^{12}.