Answers - Fractions, decimals and percentages

Year: Year 10

Answer to Question 1

a. $\frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}$ The lowest common denominator of 3 and 4 is 12, so we change both fractions into twelfths before adding. The answer is $1\frac{5}{12}$.

b. $\frac{11}{2} - \frac{11}{4} = \frac{22}{4} - \frac{11}{4} = \frac{11}{4} = 2\frac{3}{4}$ Change the mixed numbers into improper fractions, then use the common denominator 4. The answer is $2\frac{3}{4}$.

c. $\frac{3 \times 10}{5 \times 21} = \frac{30}{105} = \frac{2}{7}$ Multiply the numerators and the denominators, then simplify by dividing by 15. We could also cancel first: 5 into 10 and 3 into 21. The answer is $\frac{2}{7}$.

d. $\frac{9}{4} \div \frac{3}{2} = \frac{9}{4} \times \frac{2}{3} = \frac{18}{12} = 1\frac{1}{2}$ Change to improper fractions, then multiply by the reciprocal of the second fraction. The answer is $1\frac{1}{2}$.

Answer to Question 2

a. $7 \div 40 = 0.175$ and $0.175 \times 100 =$ **17.5%** A fraction is a division, so we divide the numerator by the denominator, then multiply by 100 for the percentage.

b. $0.36 = \frac{36}{100} = \frac{9}{25}$, and as a percentage **36%** The decimal 0.36 means 36 hundredths, and we simplify by dividing top and bottom by 4.

c. $62.5\% = 0.625 = \frac{625}{1000} = \frac{5}{8}$ Divide the percentage by 100 to get the decimal, then simplify $\frac{625}{1000}$ by dividing by 125.

d. $3 \div 11 = 0.272727\ldots = 0.\dot{2}\dot{7}$, which is **27.3%** (1 d.p.) The digits 27 repeat for ever, so we put a dot over the 2 and over the 7.

Answer to Question 3

a. 10% $= 8.40$, 5% $= 4.20$, so 15% $=$ **£12.60** Find 10% by dividing by 10, halve it for 5%, then add. 15% of £84 is £12.60.

b. 12.5% is $\frac{1}{8}$: $640 \div 8 =$ **80** 12.5% is half of 25%, which is $\frac{1}{8}$. So 12.5% of 640 is 80.

c. 1% $= 3.5$, so 7% $=$ **24.5** Find 1% by dividing by 100, then multiply by 7. 7% of 350 is 24.5.

d. 10% $= 0.24$, so 30% $= 0.72$ and 5% $= 0.12$; total $=$ **0.84 kg** Build 35% as 30%

plus 5%. 35% of 2.4 kg is 0.84 kg.

Answer to Question 4

a. Multiplier 1.08: $60 \times 1.08 =$ **£64.80** An 8% increase means we keep 100% and add 8%, so the multiplier is $1 + 0.08 = 1.08$. The new value is £64.80.
b. Multiplier 0.85: $250 \times 0.85 =$ **212.5 kg** A 15% decrease leaves $100\% - 15\% = 85\%$, so the multiplier is 0.85. The new value is 212.5 kg.
c. Multiplier 1.025: $840 \times 1.025 =$ **861 m** An increase of 2.5% gives the multiplier $1 + 0.025 = 1.025$. The new value is 861 m.
d. Multiplier 0.996: $12\,500 \times 0.996 =$ **£12 450** A decrease of 0.4% leaves 99.6%, so the multiplier is 0.996. The new value is £12 450.

Answer to Question 5

a. $\frac{20}{80} = 0.25 =$ **25% increase** Percentage change $= \frac{\text{change}}{\text{original}} \times 100$. The change is $100 - 80 = 20$, so this is a 25% increase.
b. $\frac{60}{250} = 0.24 =$ **24% decrease** The value falls by $250 - 190 = 60$. Always divide by the original value, 250. This is a 24% decrease.
c. $\frac{9}{36} = 0.25 =$ **25% increase** The change is $45 - 36 = 9$, and we divide by the original value 36. This is a 25% increase.
d. $\frac{0.5}{2.5} = 0.2 =$ **20% decrease** The change is $2.5 - 2 = 0.5$, divided by the original 2.5. This is a 20% decrease.

Answer to Question 6

a. $54 \div 1.2 =$ **£45** The new price is 120% of the original, so we divide by the multiplier 1.2 to reverse the change. The original price was £45.
b. $68 \div 0.85 =$ **£80** After a 15% decrease the price is 85% of the original, so we divide by 0.85. The original price was £80.
c. $175.50 \div 1.08 =$ **£162.50** The value is 108% of the original, so we divide by 1.08. The original value was £162.50.
d. $91 \div 0.65 =$ **140 g** After a 35% decrease the mass is 65% of the original, so we divide by 0.65. The original mass was 140 g.

Answer to Question 7

a. $480 \div 1.2 =$ **£400** The price including VAT is 120% of the price without VAT, so we

divide by 1.2.
b. $36 \div 1.2 = 30$ without VAT, so VAT $= 36 - 30 =$ **£6**
c. $27 \times 0.85 =$ **£22.95** A 15% reduction leaves 85% of the price. The lamp costs £22.95 in the sale.
d. The 20% is added to the price *without* VAT (£400), so the VAT is 20% of £400 $=$ £80. 20% of £480 is wrong because £480 already includes VAT.

Answer to Question 8

a. $120 \times 0.75 =$ **£90** A 25% reduction leaves 75% of the price, so the multiplier is 0.75. The sale price is £90.
b. $90 \times 0.9 =$ **£81** The voucher takes 10% off the sale price, not the original price, so the multiplier is 0.9. The coat costs £81.
c. $0.75 \times 0.9 = 0.675$, so the overall reduction is $1 - 0.675 =$ **32.5%**. Multiplying the two multipliers gives one multiplier for both changes. The overall saving is 32.5%, not 35%.
d. $120 \times 0.65 = £78$. The second customer pays **£3 less** $(81 - 78)$. The second customer pays £78, so the second customer pays less, by £3.

Answer to Question 9

a. Taxable income $= 28\,000 - 12\,570 = 15\,430$. Tax $= 0.2 \times 15\,430 =$ **£3086** Only the income above the personal allowance is taxed, so we subtract £12 570 first. Priya pays £3086 tax a year.
b. $28\,000 - 3086 =$ **£24 914** a year, and $24\,914 \div 12 =$ **£2076.17** a month. Subtract the tax from her salary, then divide by 12 for one month.
c. $28\,000 \times 1.035 =$ **£28 980** A 3.5% rise gives the multiplier 1.035. Her new salary is £28 980.
d. New taxable income $= 28\,980 - 12\,570 = 16\,410$. New tax $= 3282$. Increase $= 3282 - 3086 =$ **£196**. The new tax is $0.2 \times 16\,410 = 3282$. Her yearly tax increases by £196.

Answer to Question 10

a. $18\,000 \times 0.85 =$ **£15 300** A 15% fall leaves 85% of the value, so the multiplier is 0.85. The car is worth £15 300 after 1 year.
b. $18\,000 \times 0.85^3 =$ **£11 054** (11 054.25) The value falls by 15% each year, so we use the multiplier 0.85 three times. The car is worth about £11 054.
c. Year 4: $18\,000 \times 0.85^4 = 9396$. Year 5: $18\,000 \times 0.85^5 = 7987$. So **5 years**. Keep multiplying by 0.85 until the value drops below £8000. The value is first below £8000 after 5 years.
d. Multiplier $0.85^3 = 0.614125$. Loss $= 1 - 0.614125 = 0.3859$, so **38.6%**. The car keeps 61.4% of its value, so it loses about 38.6%. This is not $3 \times 15\% = 45\%$,

because each fall is worked out on a smaller value.

Answer to Question 11

a. $42\,000 \times 1.024^5 = 47\,288$, so **47 300**.
b. $42\,000 \times 1.024^7 = 49\,585$, which is below 50 000, but $42\,000 \times 1.024^8 = 50\,775$. So **8 years**.
c. $42\,000 \times 0.024 = 1008 \approx 1000$. The first year's increase is 2.4% of 42 000.
d. Compare the two populations year by year. Year 14: $58\,539$ and $58\,265$ (first town still bigger). Year 15: $59\,944$ and $60\,071$ (second town bigger). So the second town overtakes after **15 years**.

Answer to Question 12

a. Ayo $\frac{54}{72} = 75\%$. Bella $\frac{45}{72} = 62.5\%$. Carlos $\frac{63}{72} = 87.5\%$. Divide each score by 72 and multiply by 100.
b. Ayo $\frac{63}{90} = 70\%$. Bella $\frac{72}{90} = 80\%$. Carlos $\frac{54}{90} = 60\%$. Divide each score by 90 and multiply by 100.
c. Ayo $0.6 \times 75 + 0.4 \times 70 = 45 + 28 =$ **73%**. Bella $0.6 \times 62.5 + 0.4 \times 80 = 37.5 + 32 =$ **69.5%**. Carlos $0.6 \times 87.5 + 0.4 \times 60 = 52.5 + 24 =$ **76.5%**. The final grade is a weighted mean: Paper 1 counts more because its weight is 60%.
d. **Carlos** has the highest final percentage (76.5%). Carlos has the highest final percentage, because the paper with the bigger weight was his best paper.

Answer to Question 13

a. Walk: $\frac{2}{5} \times 360 =$ **144**. Bus: $\frac{1}{4} \times 360 =$ **90**.
b. $1 - \frac{2}{5} - \frac{1}{4} = \frac{20-8-5}{20} = \frac{7}{20} = 35\%$. The fractions must add up to 1 for the whole group, so subtract the other two fractions.
c. Car or bike: $360 - 144 - 90 = 126$. Cycling: $0.4 \times 126 =$ **50.4**, so about **50 pupils** (the data must be rounded).
d. Walk 144°, bus 90°, car or bike 126°. Check: $144 + 90 + 126 = 360$.

Answer to Question 14

a. Interest each year $= 0.03 \times 2000 = 60$. Over 4 years $= 240$. Total = **£2240**. Simple interest is the same every year because it is always worked out on the original £2000.

b. $2000 \times 1.03^4 =$ **£2251.02** Compound interest uses the multiplier 1.03 once for each of the 4 years.
c. $2251.02 - 2240 =$ **£11.02** So Jay has £11.02 more than Sam.
d. With compound interest the interest earned is added to the account, so the next year's interest is calculated on a larger amount.

Answer to Question 15

a. $0.9 \times 0.75 =$ **0.675** Taking 10% off gives the multiplier 0.9 and taking 25% off gives 0.75. Multiply them to get one multiplier for both discounts.
b. $87.75 \div 0.675 =$ **£130** Dev's price is 0.675 of the original, so we divide by 0.675 to reverse the change. The marked price was £130.
c. $1 - 0.675 = 0.325$, so **32.5%** off. The overall multiplier leaves 67.5% of the price, so the reduction is 32.5%.
d. 35% off would give $130 \times 0.65 = £84.50$, which is less than £87.75. The 25% is taken from the already reduced price, so the overall discount is only 32.5%.

Answer to Question 16

a. $x = 0.777\ldots$, $10x = 7.777\ldots$. Subtract: $9x = 7$, so $x = \frac{7}{9}$. The repeating part has one digit, so we multiply by 10 to line up the repeats. Subtracting removes the endless decimal part.
b. $x = 0.4545\ldots$, $100x = 45.4545\ldots$. Subtract: $99x = 45$, so $x = \frac{45}{99} = \frac{5}{11}$. The repeating part has two digits, so we multiply by 100. Then divide 45 and 99 by their HCF, 9.
c. $x = 0.2111\ldots$, $10x = 2.111\ldots$, $100x = 21.111\ldots$. Subtract: $90x = 19$, so $x = \frac{19}{90}$. Only the 1 repeats, so we use $100x$ and $10x$, which both end in $.111\ldots$. The fraction $\frac{19}{90}$ is already in its simplest form.
d. $x = 0.999\ldots$, $10x = 9.999\ldots$, so $9x = 9$ and $x = 1$. So $0.\dot{9}$ and 1 are two ways of writing the same number.

Answer to Question 17

a. $1000 \times 1.07^{10} =$ **£1967.15** 7% compound interest gives the multiplier 1.07, used 10 times. The account is worth £1967.15.
b. $1.07^{10} = 1.967 < 2$ and $1.07^{11} = 2.105 > 2$. So **11 years**. After 10 years the multiplier is 1.967, just under 2, and after 11 years it is $1.07^{11} = 2.105$, more than 2. So the amount first doubles after 11 years.
c. $72 \div 7 = 10.3$ years, close to the exact answer of about 10.2 years (between 10

and 11). The rule of 72 is a quick estimate, and here it agrees well with the exact calculation.

d. $1.04^{17} = 1.948$ and $1.04^{18} = 2.026$. So **18 years**. Try powers of 1.04 until the result first passes 2. It takes 18 years to double at 4%.

Answer to Question 18

a. $1 - \frac{1}{3} = \frac{2}{3}$ After spending $\frac{1}{3}$, Lena has $\frac{2}{3}$ of her money left.

b. $\frac{2}{5} \times \frac{2}{3} = \frac{4}{15}$ She spends $\frac{2}{5}$ of what is left, so we multiply $\frac{2}{5}$ by $\frac{2}{3}$.

c. $\frac{2}{3} - \frac{4}{15} = \frac{10-4}{15} = \frac{6}{15} = \frac{2}{5}$ Subtract the lunch from what was left after the book, using the common denominator 15.

d. $\frac{2}{5}$ of the original is £12, so the original is $12 \div \frac{2}{5} = 12 \times \frac{5}{2} =$ **£30**. The book cost $\frac{1}{3} \times 30 =$ **£10**. So Lena started with £30 and the book cost £10. Check: $30 - 10 = 20$, lunch $= \frac{2}{5} \times 20 = 8$, and $20 - 8 = 12$.

Answer to Question 19

a. Profit $= 330 - 240 = 90$. $\frac{90}{240} = 0.375 =$ **37.5%** Percentage profit is the profit divided by the cost price, times 100. The percentage profit is 37.5%.

b. $275 \times 1.4 =$ **£385** A 40% profit means selling at 140% of the cost price, so the multiplier is 1.4. The selling price is £385.

c. $399 \div 1.33 =$ **£300** The selling price is 133% of the cost price, so we divide by 1.33. The cost price was £300.

d. $192 \div 0.8 =$ **£240** A 20% loss means the selling price is 80% of the cost price, so we divide by 0.8. The cost price was £240.

Answer to Question 20

a. $\left(1 + \frac{x}{100}\right)\left(1 - \frac{x}{100}\right) = 1 - \frac{x^2}{10\,000}$ (difference of two squares).

b. $1 - \frac{x^2}{10\,000} = 0.91$, so $x^2 = 900$ and $x =$ **30**. A price 9% lower means the multiplier is 0.91, so $\frac{x^2}{10\,000} = 0.09$. We take the positive root, so the percentage is 30%.

c. $200 \times 1.3 = 260$ and $260 \times 0.7 = 182$. $\frac{18}{200} = 9\%$ lower ✓.

d. The multiplier $1 - \frac{x^2}{10\,000}$ is always less than 1 for $x > 0$, so the final price is always **lower**.