Worksheet - Formulae and identities

Year: Year 10 · 20

Show your working.

Question 1

Question 1 – Substituting into formulae *(Section A · Skills practice)*

Calculator allowed. Use the given values to work out each answer.

a. $v = u + at$ with $u = 5$, $a = 2.5$, $t = 6$. Find v.
b. $s = ut + \frac{1}{2}at^2$ with $u = 3$, $a = 4$, $t = 5$. Find s.
c. $A = \frac{1}{2}(a + b)h$ with $a = 7$, $b = 11$, $h = 6$. Find A.
d. $v^2 = u^2 + 2as$ with $u = 4$, $a = 3$, $s = 10$. Find v (positive value).

Question 2

Question 2 – Changing the subject: one step *(Section A · Skills practice)*

Make the letter in brackets the subject of each formula.

a. $y = mx + c$ (make x the subject)
b. $P = 2l + 2w$ (make l the subject)
c. $V = lwh$ (make h the subject)
d. $C = 2\pi r$ (make r the subject)

Question 3

Question 3 – Changing the subject: harder *(Section A · Skills practice)*

Make the letter in brackets the subject. Show each step.

a. $A = \frac{1}{2}(a + b)h$ (make a the subject)
b. $v^2 = u^2 + 2as$ (make a the subject)
c. $s = \frac{1}{2}(u + v)t$ (make v the subject)
d. $d = \sqrt{2h}$ (make h the subject)

Question 4

Question 4 – Expression, equation, formula or identity? *(Section A · Skills practice)*

Decide whether each is an expression, an equation, a formula or an identity. Give a reason.

a. $3x + 5 = 20$

b. $A = \pi r^2$
c. $(x + 2)(x - 2) \equiv x^2 - 4$
d. $4a - 7b + 2$

Question 5

Question 5 – Showing that two expressions are identical *(Section A · Skills practice)*

Show that each identity is true by expanding and simplifying one side.

a. $2(x + 4) + 3(x - 1) \equiv 5x + 5$
b. $(x + 3)^2 - (x - 3)^2 \equiv 12x$
c. $(x + 5)(x - 2) \equiv x^2 + 3x - 10$
d. $(2x - 1)^2 \equiv 4x^2 - 4x + 1$

Question 6

Question 6 – Making x the subject with squares *(Section A · Skills practice)*

Make x the subject of each formula. Give both positive and negative roots where they apply.

a. $y = 3x^2 - 5$
b. $y = \frac{x^2}{4} + 1$
c. $A = \pi x^2$ (positive x only)
d. $y = (x + 2)^2$

Question 7

Question 7 – Temperature conversion *(Section B · Real-life practice)*

In the UK, weather forecasts use degrees Celsius (°C). Some visitors use degrees Fahrenheit (°F). The formula is $F = 1.8C + 32$.

City	Temperature
London	25 °C
Edinburgh	11 °C
Cardiff	-5 °C

a. Convert each temperature to °F.
b. Make C the subject of the formula.
c. A US visitor's oven is set to 350 °F. Find the temperature in °C to the nearest degree.
d. At what temperature are the Celsius and Fahrenheit values equal?

Question 8

Question 8 – Energy bills *(Section B · Real-life practice)*

An energy company charges £C per month by the formula $C = 0.28u + 0.45d$ where u is the number of units of electricity used and d is the number of days. The standing charge is built into the second term.

a. Work out the bill for 160 units in a 30-day month.
b. Make u the subject of the formula.
c. A customer in a 31-day month has a bill of £70. Find the units used, to the nearest unit.
d. Another firm charges £$C = 0.25u + 0.62d$. For 160 units over 30 days, which firm is cheaper, and by how much?

Question 9

Question 9 – The Sunday roast *(Section B · Real-life practice)*

The cooking time T minutes for a joint of beef of mass w kg is given by $T = 40w + 20$.

a. Work out the cooking time for a 2.5 kg joint, in hours and minutes.
b. Make w the subject of the formula.
c. Dinner is planned for 6:30 pm. The joint weighs 3.2 kg. At what time should it go in the oven?
d. What is the largest joint that can be cooked in $2\frac{1}{2}$ hours?

Question 10

Question 10 – Dropping from the bridge *(Section B · Real-life practice)*

A stone is dropped from a bridge. The distance fallen, s metres, after t seconds is $s = \frac{1}{2}gt^2$ where $g = 9.8$ m/s².

a. How far does the stone fall in 3 seconds?
b. Make t the subject of the formula.
c. The bridge is 78.4 m above the river. How long does the stone take to reach the water?
d. The stone takes 2 seconds to reach the water from a different bridge. How high is that bridge?

Question 11

Question 11 – Body mass index *(Section B · Real-life practice)*

Body mass index is $B = \dfrac{m}{h^2}$ where m is mass in kg and h is height in metres. A school nurse uses it in a health talk.

a. Work out B for $m = 70$ and $h = 1.75$, to 1 decimal place.

b. Make m the subject.
c. Find the mass for $B = 24$ and $h = 1.6$.
d. Make h the subject of the formula and find h when $m = 63$ and $B = 25$.

Question 12

Question 12 – Compound interest *(Section B · Real-life practice)*

The value of a savings account after n years is $A = P\left(1 + \frac{r}{100}\right)^n$, where P is the amount invested and r is the yearly percentage rate. Calculator allowed.

a. Find A when $P = 500$, $r = 4$ and $n = 3$, to the nearest penny.
b. Find A when $P = 1200$, $r = 2.5$ and $n = 5$, to the nearest penny.
c. Simple interest is $I = \frac{PRT}{100}$. Make T the subject.
d. Find the time for £800 to earn £120 simple interest at 3% per year.

Question 13

Question 13 – Pool depth *(Section B · Real-life practice)*

The cross-section of a swimming pool is a trapezium with parallel sides a (the shallow end) and b (the deep end) and a length h (the length of the pool). The area of the cross-section is $A = \frac{1}{2}(a + b)h$.

a. Find the area when $a = 1$ m, $b = 2.5$ m and $h = 20$ m.
b. Make b the subject of the formula.
c. The area is 48 m² with $a = 1.2$ m and $h = 20$ m. Find b.
d. The pool's width is 8 m. Find the volume of water in the pool in the situation from part (a).

Question 14

Question 14 – The paint tin *(Section B · Real-life practice)*

A cylindrical paint tin has volume $V = \pi r^2 h$.

a. Find V for $r = 7$ cm and $h = 10$ cm, to the nearest cm³.
b. Make r the subject of the formula.
c. A tin must hold exactly 750 cm³ and has height 12 cm. Find r to 1 decimal place.
d. Make h the subject and find h for a tin with radius 6 cm and volume 1000 cm³, to 1 d.p.

Question 15

Question 15 – Higher: the subject appears twice *(Section C · Problem solving)*

Higher tier. When the new subject appears in more than one place, collect all the terms containing it on one side, then factorise.

a. Make x the subject of $ax + b = cx + d$.
b. Make x the subject of $y = \dfrac{2x + 1}{x - 3}$.
c. Make a the subject of $S = \dfrac{a}{1 - r}$ (r is not 1).
d. Make p the subject of $p(q + 1) = 3(p - 2)$.

Question 16

Question 16 – Proving identities and statements *(Section C · Problem solving)*

Use algebra to prove each statement. Remember that you must not use numbers as proof, only as a check.

a. Prove that $(n + 1)^2 - (n - 1)^2 \equiv 4n$.
b. Prove that the sum of the squares of two consecutive whole numbers is always odd.
c. Prove that $(2n + 1)^2 - 1$ is always a multiple of 4. (Hint: expand and factorise.)
d. Show that $(2n + 1)^2 - 1$ is actually a multiple of 8.

Question 17

Question 17 – Higher: stopping distances *(Section C · Problem solving)*

Higher tier. The stopping distance d metres of a car travelling at v miles per hour is modelled by $d = \dfrac{v^2}{60} + 0.3v$ (this is a simplified model).

a. Find the stopping distance at 30 mph.
b. Find the stopping distance at 70 mph, to the nearest metre.
c. Show that $d = 60$ gives $v^2 + 18v - 3600 = 0$.
d. Solve this equation to find the speed that gives a stopping distance of 60 m, to 1 decimal place.

Question 18

Question 18 – Finding unknown numbers in an identity *(Section C · Problem solving)*

An identity is true for all values of x, so the coefficients on each side must match.

a. Find a and b if $(x + a)^2 + b \equiv x^2 + 6x + 13$.
b. Find p and q if $p(x + 3) + q \equiv 5x + 19$.
c. Find k if $(x + k)(x - 4) \equiv x^2 + 2x - 24$.
d. Show that there are no values of a and b such that $2(x + a) \equiv 2x + b + x$ for all x.

Question 19

Question 19 – Rearranging in context *(Section C · Problem solving)*

The formula for the length L of an arc of a circle of radius r and angle θ degrees is $L = \dfrac{\theta}{360} \times 2\pi r$.

a. Make θ the subject.
b. Make r the subject.
c. Find θ when $L = 15.7$ and $r = 10$, to the nearest degree.
d. Find r when $L = 22$ and $\theta = 90$, to 1 decimal place.

Question 20

Question 20 – Challenge: triangular numbers *(Section D · Challenge)*

The nth triangular number is $T_n = \dfrac{n(n+1)}{2}$. The first few are 1, 3, 6, 10, 15 (dots arranged in a triangle).

a. Find T_6 and T_{10}.
b. Show that $T_n + T_{n+1} = (n+1)^2$.
c. Use part (b) to explain why the sum of two consecutive triangular numbers is always a square number.
d. Find n if $T_n = 120$.