Answers - Formulae and identities

Year: Year 10

Answer to Question 1

a. $v = 5 + 2.5 \times 6 = 5 + 15 =$ **20** Substitute the values, then multiply before adding (BIDMAS). So $v = 20$.

b. $s = 3 \times 5 + \frac{1}{2} \times 4 \times 25 = 15 + 50 =$ **65** Square t first: $t^2 = 25$. Then $\frac{1}{2} \times 4 \times 25 = 50$, so $s = 65$.

c. $A = \frac{1}{2} \times 18 \times 6 =$ **54** Add the parallel sides first because they are in a bracket: $7 + 11 = 18$. So $A = 54$.

d. $v^2 = 16 + 2 \times 3 \times 10 = 76$, so $v = \sqrt{76} =$ **8.72** (3 s.f.) Find v^2 first, then take the positive square root because the question asks for the positive value. So $v = 8.72$ to 3 significant figures.

Answer to Question 2

a. $y - c = mx$, so $x = \dfrac{y - c}{m}$ Subtract c from both sides, then divide both sides by m. The new subject is $x = \dfrac{y - c}{m}$.

b. $P - 2w = 2l$, so $l = \dfrac{P - 2w}{2}$ Subtract $2w$ from both sides, then divide by 2. So $l = \dfrac{P - 2w}{2}$.

c. $h = \dfrac{V}{lw}$ Here h is multiplied by lw, so we divide both sides by lw. So $h = \dfrac{V}{lw}$.

d. $r = \dfrac{C}{2\pi}$ Here r is multiplied by 2π, so we divide both sides by 2π. So $r = \dfrac{C}{2\pi}$.

Answer to Question 3

a. $2A = (a + b)h$, $\dfrac{2A}{h} = a + b$, so $a = \dfrac{2A}{h} - b$ Undo the operations in reverse order: multiply by 2, divide by h, then subtract b.

b. $v^2 - u^2 = 2as$, so $a = \dfrac{v^2 - u^2}{2s}$ Subtract u^2 from both sides, then divide by $2s$.

c. $2s = (u + v)t$, $\dfrac{2s}{t} = u + v$, so $v = \dfrac{2s}{t} - u$ Multiply by 2, divide by t, then subtract u from both sides.

d. Square both sides: $d^2 = 2h$, so $h = \dfrac{d^2}{2}$ Squaring undoes the square root, then

dividing by 2 isolates h.

Answer to Question 4

a. An **equation**: it is true only for $x = 5$.
b. A **formula**: it links variables, here the area and radius of a circle.
c. An **identity**: it is true for every value of x (the sign $\equiv$ shows this).
d. An **expression**: it has no equals sign.

Answer to Question 5

a. $2x + 8 + 3x - 3 = 5x + 5$ ✓ Expand both brackets and collect like terms: $2x + 3x = 5x$ and $8 - 3 = 5$. The left-hand side equals the right-hand side, so the identity is true.
b. $(x^2 + 6x + 9) - (x^2 - 6x + 9) = 12x$ ✓ The bracket in front of the second square changes every sign: the x^2 terms cancel and so do the 9s, leaving $6x + 6x = 12x$. So the identity is true.
c. $x^2 - 2x + 5x - 10 = x^2 + 3x - 10$ ✓ Multiply each term in the first bracket by each term in the second, then collect $-2x + 5x = 3x$. So the identity is true.
d. $(2x - 1)(2x - 1) = 4x^2 - 2x - 2x + 1 = 4x^2 - 4x + 1$ ✓ A squared bracket is the bracket times itself, so there are two middle terms of $-2x$. So the identity is true for every value of x.

Answer to Question 6

a. $y + 5 = 3x^2$, $x^2 = \dfrac{y+5}{3}$, so $x = \pm\sqrt{\dfrac{y+5}{3}}$ Add 5, divide by 3, then take the square root. A square root can be positive or negative, so we write $\pm$.
b. $y - 1 = \dfrac{x^2}{4}$, $x^2 = 4(y-1)$, so $x = \pm 2\sqrt{y-1}$ Subtract 1, multiply by 4, then take the square root; $\sqrt{4} = 2$ comes out of the root.
c. $x^2 = \dfrac{A}{\pi}$, so $x = \sqrt{\dfrac{A}{\pi}}$ Divide by π, then take the positive square root because x is a length.
d. $\sqrt{y} = \pm(x + 2)$, so $x = -2 \pm \sqrt{y}$ Take the square root of both sides first, then subtract 2 from both sides.

Answer to Question 7

a. London: $1.8 \times 25 + 32 =$ **77 °F**. Edinburgh: $1.8 \times 11 + 32 =$ **51.8 °F**. Cardiff:

$1.8 \times (-5) + 32 =$ **23 °F**.

b. $F - 32 = 1.8C$, so $C = \dfrac{F - 32}{1.8}$. Undo the operations in reverse order: subtract 32 first, then divide by 1.8.

c. $C = \dfrac{350 - 32}{1.8} = 176.7$, so **177 °C**. So an oven at 350 °F is at about 177 °C.

d. Set $C = F$: $C = 1.8C + 32$, so $-0.8C = 32$ and $C =$ **−40**. At -40 degrees the two scales agree.

Answer to Question 8

a. $0.28 \times 160 + 0.45 \times 30 = 44.80 + 13.50 =$ **£58.30** Substitute $u = 160$ and $d = 30$. The bill is £58.30.

b. $C - 0.45d = 0.28u$, so $u = \dfrac{C - 0.45d}{0.28}$ Subtract $0.45d$ from both sides, then divide by 0.28.

c. $u = \dfrac{70 - 0.45 \times 31}{0.28} = \dfrac{56.05}{0.28} = 200.2$, so **200 units**. Substitute $C = 70$ and $d = 31$ into the rearranged formula. The customer used about 200 units.

d. Other firm: $0.25 \times 160 + 0.62 \times 30 = 40 + 18.60 = 58.60$. The first firm is cheaper by $58.60 - 58.30 =$ **£0.30**. So the first firm is cheaper, by 30p.

Answer to Question 9

a. $T = 40 \times 2.5 + 20 = 120$ minutes $=$ **2 hours**. Substitute $w = 2.5$. 120 minutes is 2 hours, so the joint cooks for 2 hours.

b. $T - 20 = 40w$, so $w = \dfrac{T - 20}{40}$. Subtract 20 from both sides, then divide by 40.

c. $T = 40 \times 3.2 + 20 = 148$ minutes $= 2$ h 28 min. Back from 18:30: **16:02** (about 4:02 pm). Count back 2 hours from 18:30 to 16:30, then 28 more minutes to 16:02. The joint should go in at about 4:02 pm.

d. $2\frac{1}{2}$ hours $= 150$ min. $w = \dfrac{150 - 20}{40} =$ **3.25 kg**. Change the time into minutes, then use the rearranged formula. The largest joint is 3.25 kg.

Answer to Question 10

a. $s = \frac{1}{2} \times 9.8 \times 9 =$ **44.1 m**. Square the time first: $t^2 = 9$. The stone falls 44.1 m in 3 seconds.

b. $2s = gt^2$, $t^2 = \dfrac{2s}{g}$, so $t = \sqrt{\dfrac{2s}{g}}$. Multiply by 2, divide by g, then take the positive square root because time is positive.

c. $t = \sqrt{\dfrac{2 \times 78.4}{9.8}} = \sqrt{16} =$ **4 s**. Substitute $s = 78.4$ and $g = 9.8$ into the rearranged formula. The stone takes 4 seconds to reach the water.
d. $s = \frac{1}{2} \times 9.8 \times 4 =$ **19.6 m**. Substitute $t = 2$, so $t^2 = 4$. The bridge is 19.6 m high.

Answer to Question 11

a. $B = 70 \div 1.75^2 = 70 \div 3.0625 =$ **22.9** Square the height first: $1.75^2 = 3.0625$. The BMI is 22.9 to 1 decimal place.
b. $m = Bh^2$ Multiply both sides by h^2. So $m = Bh^2$.
c. $m = 24 \times 1.6^2 = 24 \times 2.56 =$ **61.44 kg** Substitute into $m = Bh^2$. The mass is 61.44 kg.
d. $h^2 = \dfrac{m}{B}$, so $h = \sqrt{\dfrac{m}{B}}$. $h = \sqrt{63 \div 25} = \sqrt{2.52} =$ **1.59 m**. Multiply by h^2, divide by B, then take the positive square root because height is positive. The height is 1.59 m.

Answer to Question 12

a. $500 \times 1.04^3 =$ **£562.43** The multiplier for 4% growth is 1.04, and it is applied once for each of the 3 years. The account is worth £562.43.
b. $1200 \times 1.025^5 =$ **£1357.69** The multiplier for 2.5% is 1.025, applied 5 times. The account is worth £1357.69.
c. $100I = PRT$, so $T = \dfrac{100I}{PR}$ Multiply both sides by 100, then divide by PR.
d. $T = \dfrac{100 \times 120}{800 \times 3} = \dfrac{12\,000}{2400} =$ **5 years**. Substitute into the rearranged formula. It takes 5 years to earn £120.

Answer to Question 13

a. $A = \frac{1}{2} \times 3.5 \times 20 =$ **35 m²**. Add the parallel sides, halve, then multiply by the length. The area is 35 m².
b. $2A = (a + b)h$, so $\dfrac{2A}{h} = a + b$ and $b = \dfrac{2A}{h} - a$. Multiply by 2, divide by h, then subtract a.
c. $b = \dfrac{96}{20} - 1.2 = 4.8 - 1.2 =$ **3.6 m**. Substitute $A = 48$, $h = 20$ and $a = 1.2$ into the rearranged formula. The deep end is 3.6 m deep.
d. Volume $= 35 \times 8 =$ **280 m³**. The pool is a prism, so its volume is the cross-section area times the width. The pool holds 280 m³ of water.

Answer to Question 14

a. $\pi \times 49 \times 10 =$ **1539 cm³** Square the radius first: $7^2 = 49$. The volume is about 1539 cm³.

b. $r^2 = \dfrac{V}{\pi h}$, so $r = \sqrt{\dfrac{V}{\pi h}}$ Divide by πh, then take the positive square root because r is a length.

c. $r = \sqrt{\dfrac{750}{12\pi}} = \sqrt{19.89} =$ **4.5 cm** Substitute $V = 750$ and $h = 12$. The radius is 4.5 cm.

d. $h = \dfrac{V}{\pi r^2} = \dfrac{1000}{36\pi} =$ **8.8 cm** Divide both sides by πr^2. The tin is 8.8 cm tall.

Answer to Question 15

a. $ax - cx = d - b$, $x(a - c) = d - b$, so $x = \dfrac{d - b}{a - c}$ Collect the x terms on one side and the other terms on the other, then factorise out x and divide by $a - c$.

b. $y(x - 3) = 2x + 1$, $xy - 3y = 2x + 1$, $xy - 2x = 3y + 1$, $x(y - 2) = 3y + 1$, so $x = \dfrac{3y + 1}{y - 2}$ Multiply both sides by $x - 3$ to clear the fraction, then collect the x terms on the left. Factorise and divide by $y - 2$.

c. $a = S(1 - r)$ Multiply both sides by $1 - r$. This is allowed because r is not 1, so $1 - r$ is not zero.

d. $pq + p = 3p - 6$, $pq + p - 3p = -6$, $p(q - 2) = -6$, so $p = \dfrac{-6}{q - 2} = \dfrac{6}{2 - q}$ Expand the bracket, collect the p terms on one side, factorise and divide by $q - 2$.

Answer to Question 16

a. $(n^2 + 2n + 1) - (n^2 - 2n + 1) = 4n$. ✓

b. $n^2 + (n + 1)^2 = 2n^2 + 2n + 1 = 2(n^2 + n) + 1$, which is 2 times a whole number plus 1: **odd**.

c. $(2n + 1)^2 - 1 = 4n^2 + 4n + 1 - 1 = 4n^2 + 4n = 4n(n + 1)$, a multiple of **4**.

d. $4n(n + 1)$: one of n and $n + 1$ is even, so $n(n + 1)$ is even $= 2k$, and $4 \times 2k = 8k$. So it is a multiple of **8**.

Answer to Question 17

a. $d = \dfrac{900}{60} + 9 = 15 + 9 =$ **24 m**. Substitute $v = 30$: $30^2 = 900$ and $0.3 \times 30 = 9$. The stopping distance is 24 m.

b. $d = \frac{4900}{60} + 21 = 81.67 + 21 = 102.67$, so **103 m**. Substitute $v = 70$: $70^2 = 4900$. The stopping distance is about 103 m.

c. $\frac{v^2}{60} + 0.3v = 60$. Multiply by 60: $v^2 + 18v = 3600$, so $v^2 + 18v - 3600 = 0$. Multiplying every term by 60 clears the fraction, then subtract 3600 from both sides.

d. $v = \frac{-18 + \sqrt{18^2 + 4 \times 3600}}{2} = \frac{-18 + \sqrt{14724}}{2} = \frac{-18 + 121.35}{2} =$ **51.7 mph**. Use the quadratic formula with $a = 1$, $b = 18$, $c = -3600$. We reject the negative root because a speed cannot be negative.

Answer to Question 18

a. Expand: $x^2 + 2ax + a^2 + b$. Compare x terms: $2a = 6$, so $a = 3$. Constant: $9 + b = 13$, so $b =$ **4**. So $a =$ **3**, $b =$ **4**.

b. $px + 3p + q$. Compare x: $p =$ **5**. Constant: $15 + q = 19$, so $q =$ **4**.

c. $x^2 + (k-4)x - 4k$. Compare x: $k - 4 = 2$, so $k = 6$; check the constant: $-4 \times 6 = -24$ ✓. So $k =$ **6**.

d. The left side has $2x$ and the right side has $3x + b$. The x terms $2x$ and $3x$ cannot match, so **no values of a and b work**.

Answer to Question 19

a. $\theta = \frac{360L}{2\pi r} = \frac{180L}{\pi r}$ Simplify $\frac{360}{2} = 180$, then multiply both sides by 360 and divide by $2\pi r$.

b. $r = \frac{360L}{2\pi\theta} = \frac{180L}{\pi\theta}$ In the same way, multiply by 360 and divide by $2\pi\theta$.

c. $\theta = \frac{180 \times 15.7}{\pi \times 10} = \frac{2826}{31.416} = 89.96$, so **90°**. Substitute $L = 15.7$ and $r = 10$. The angle is 90° to the nearest degree.

d. $r = \frac{180 \times 22}{\pi \times 90} = \frac{3960}{282.74} =$ **14.0 cm**. Substitute $L = 22$ and $\theta = 90$. The radius is 14.0 cm to 1 decimal place.

Answer to Question 20

a. $T_6 = \frac{6 \times 7}{2} =$ **21** and $T_{10} = \frac{10 \times 11}{2} =$ **55**.

b. $\frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2} = \frac{(n+1)(n+n+2)}{2} = \frac{(n+1)(2n+2)}{2} = (n+1)^2$.

Write both fractions over 2, then take out the common factor $(n+1)$.

c. The result $(n+1)^2$ is the square of a whole number, so it is a **square number**. For example $6+10=16=4^2$ and $10+15=25=5^2$.

d. $n(n+1)=240$, so $n^2+n-240=0$, $(n-15)(n+16)=0$ and $n=$ **15**.