Test answers - Factors, multiples, primes and HCF/LCM

Year: Year 10 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. **1, 2, 3, 5, 6, 10, 15, 30** (2 marks; 1 if one error or omission) Work in factor pairs: 1×30, 2×15, 3×10, 5×6. So the factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.
b. **11, 13, 17, 19, 23, 29** (2 marks; 1 if one error or omission) A prime has exactly two factors. The numbers 15, 21, 25 and 27 are not prime because they have other factors. The primes are 11, 13, 17, 19, 23 and 29.

Answer to Question 2 (4 marks)

a. $420 = 2^2 \times 3 \times 5 \times 7$ (1 for dividing by primes, 1 for the answer) Divide by 2 twice to get 105, then by 3, 5 and 7. So $420 = 2^2 \times 3 \times 5 \times 7$.
b. $504 = 2^3 \times 3^2 \times 7$ (1). HCF $= 2^2 \times 3 \times 7 =$ **84**, LCM $= 2^3 \times 3^2 \times 5 \times 7 =$ **2520** (1 for both). For the HCF take the lowest power of each common prime; for the LCM take the highest power of every prime. The HCF is 84 and the LCM is 2520.

Answer to Question 3 (4 marks)

a. LCM of 20 and 30 is 60 (1). Next together at **08:00** (1) Using $20 = 2^2 \times 5$ and $30 = 2 \times 3 \times 5$, the LCM is $2^2 \times 3 \times 5 = 60$ minutes. The buses next leave together at 08:00.
b. Every 60 minutes: 07:00, 08:00, 09:00, 10:00 (1). That is **4 times** (1). The buses leave together every 60 minutes, so they leave together 4 times between 07:00 and 10:00.

Listing multiples also works: 20, 40, 60 and 30, 60.

Answer to Question 4 (4 marks)

a. HCF of 270 and 180 (1) = **90 cm** (1) The side must divide exactly into both 270 and 180, so it is a common factor. Using $270 = 2 \times 3^3 \times 5$ and $180 = 2^2 \times 3^2 \times 5$, the HCF is $2 \times 3^2 \times 5 = 90$. The largest tile has side 90 cm.
b. $270 \div 90 = 3$ and $180 \div 90 = 2$ (1); $3 \times 2 =$ **6 tiles** (1). Three tiles fit along the length and two fit along the height, so 6 tiles are needed.

Answer to Question 5 (4 marks)

a. $10^4 = \mathbf{10\ 000}$ (1) There are 10 choices for each of the 4 digits, so there are 10 000 possible PINs.

b. $10 \times 9 \times 8 \times 7 = \mathbf{5040}$ (1) There are 10 choices for the first digit, then 9 left for the second, 8 for the third and 7 for the fourth. So 5040 PINs have 4 different digits.

c. $n + (n + 1) + (n + 2) = 3n + 3$ (1) $= 3(n + 1)$, a multiple of 3 (1). Since $n + 1$ is a whole number, $3(n + 1)$ is a multiple of 3 for every whole number n.