Worksheet - Factors, multiples, primes and HCF/LCM

Year: Year 10 · 20

Show your working.

Question 1

Question 1 – Listing factors *(Section A · Skills practice)*

Non-calculator. Write the factors of each number in pairs, then answer the last part.

a. 36
b. 48
c. Write down the common factors of 36 and 48.
d. Which of 36 and 48 has more factors, and how many more?

Question 2

Question 2 – Primes *(Section A · Skills practice)*

A prime number has exactly two factors: 1 and itself. Answer the questions.

a. List all the prime numbers between 20 and 50.
b. Show that 91 is not prime.
c. Explain why 2 is the only even prime number.
d. Find two prime numbers that add up to 40.

Question 3

Question 3 – Prime factorisation *(Section A · Skills practice)*

Write each number as a product of its prime factors. Use index notation.

a. 180
b. 252
c. 1176
d. Check part (a) by multiplying out.

Question 4

Question 4 – HCF and LCM *(Section A · Skills practice)*

Find the highest common factor (HCF) and the lowest common multiple (LCM) of each pair.

a. 24 and 36

b. 45 and 75
c. 28 and 42
d. Use prime factors for 84 and 90.

Question 5

Question 5 – Powers and roots from prime factors *(Section A · Skills practice)*

Use prime factors to find these. Non-calculator.

a. Write 576 as a product of primes, then find $\sqrt{576}$.
b. Write 1728 as a product of primes, then find $\sqrt[3]{1728}$.
c. Show that 200 is not a square number.
d. What is the smallest whole number you must multiply 200 by to get a square number?

Question 6

Question 6 – Multiples *(Section A · Skills practice)*

Write down the first five multiples of each number and answer the final parts.

a. 6
b. 8
c. The smallest number that is a multiple of both 6 and 8.
d. The smallest number divisible by 6, 8 and 10.

Question 7

Question 7 – Tiling the kitchen floor *(Section B · Real-life practice)*

A kitchen floor measures 420 cm by 300 cm. It is to be covered with identical square tiles with no cutting. The tile side in centimetres must be a whole number.

a. Find the side of the largest square tile that can be used.
b. How many tiles are needed with the largest tile?
c. Tiles of side 60 cm cost £4.80 each. Find the cost for the floor.
d. List all the tile sizes that are 10 cm or more and are factors of both lengths.

Question 8

Question 8 – Buses at the station *(Section B · Real-life practice)*

At a bus station, Bus A leaves every 12 minutes and Bus B leaves every 18 minutes. They both leave at 08:00.

a. After how many minutes will they next leave together?
b. What time is this?

c. How many times between 08:00 and 12:00 (including 08:00) do they leave together?
d. A Bus C leaves every 8 minutes and also left at 08:00. When do all three leave together again?

Question 9

Question 9 – Party bags *(Section B · Real-life practice)*

Dev has 84 chocolates and 126 sweets. He wants to make identical party bags using all the chocolates and all the sweets.

a. What is the greatest number of bags he can make?
b. How many chocolates and how many sweets are in each bag?
c. A friend gives Dev 6 more chocolates. Can he still make the same number of identical bags? Explain.
d. What are all the possible numbers of bags Dev could make in the original situation?

Question 10

Question 10 – Flashing lights *(Section B · Real-life practice)*

At a fair, three lights flash. The red light flashes every 10 seconds, the green every 15 seconds and the blue every 25 seconds. All three flash together at the start.

Light	Flashes every
Red	10 s
Green	15 s
Blue	25 s

a. After how many seconds do all three flash together again?
b. How many times do all three flash together in the first 10 minutes (not counting the start)?
c. How many times does the red light flash in 10 minutes?
d. The red and green lights only (not blue): after how many seconds do they next flash together?

Question 11

Question 11 – The school library *(Section B · Real-life practice)*

A librarian has 360 books and wants to arrange them on a shelf in equal rows. Each row must contain the same number of books, with no books left over.

a. Write 360 as a product of its prime factors.
b. Show that 360 has 24 factors by using $(3+1)(2+1)(1+1)$.
c. The librarian does not want fewer than 10 books or more than 40 in a row. List

the possible row lengths.
d. How many rows would there be for each of your answers?

Question 12

Question 12 – Running laps *(Section B · Real-life practice)*

Ali runs a lap of the school field in 40 seconds and Beth runs a lap in 50 seconds. They start together at the start line.

a. After how many seconds will they next be together at the start line?
b. How many laps will each have completed?
c. After 10 minutes, how many complete laps has each run?
d. Beth speeds up so that a lap takes 45 seconds. When will they next be together at the start?

Question 13

Question 13 – Choices and combinations *(Section B · Real-life practice)*

A café offers a lunch deal. A customer chooses one item from each group.

Group	Choices
Bread	white, brown, seeded, wrap
Filling	cheese, ham, egg, tuna, salad
Drink	water, juice, milk

a. How many different lunches are possible?
b. How many are possible if the customer refuses tuna?
c. The café adds two new fillings. How many lunches are now possible?
d. A customer wants to try a different lunch every school day. For how many weeks (5 days) can she do it with the original menu?

Question 14

Question 14 – Square numbers on the hall floor *(Section B · Real-life practice)*

A school hall has chairs set out in a square arrangement for an assembly.

a. There are 144 chairs. How many rows are there?
b. The headteacher adds a row and a column to make a bigger square. How many chairs are there now?
c. For the Year 10 assembly, 2025 chairs are arranged in a square. How many are in each row?
d. There are 200 chairs in a store. Explain why they cannot make a square, and say how many more are needed to make the next square number.

Question 15

Question 15 – Proof with odd and even numbers *(Section C · Problem solving)*

An odd number can be written as $2k+1$ and an even number as $2k$, where k is a whole number.

a. Prove that the sum of two odd numbers is always even. Use $2a+1$ and $2b+1$.
b. Prove that the product of two odd numbers is always odd.
c. Show that n^2+n is always even (try $n=3$ and $n=4$, then prove it by factorising).
d. Is the sum of an odd number and an even number always odd? Give a proof.

Question 16

Question 16 – Using prime factors *(Section C · Problem solving)*

$A=2^3\times3^2\times5$ and $B=2^2\times3^3\times7$. Leave answers in index form where sensible.

a. Find the HCF of A and B.
b. Find the LCM of A and B.
c. Show that $A\times B=\text{HCF}\times\text{LCM}$.
d. Work out the value of A, the value of B and the value of the HCF as ordinary numbers.

Question 17

Question 17 – Finding unknown numbers *(Section C · Problem solving)*

Two whole numbers have a highest common factor of 6 and a lowest common multiple of 72.

a. One of the numbers is 18. Find the other one.
b. Check that your pair has HCF 6 and LCM 72.
c. Find all the pairs of numbers with HCF 6 and LCM 72 (the numbers are different and both bigger than 6).
d. Find the smallest number divisible by 8, 12 and 15 that is also a square number.

Question 18

Question 18 – The locker puzzle *(Section C · Problem solving)*

A school has 100 lockers, numbered 1 to 100, all shut. Student 1 opens every locker. Student 2 changes (opens or shuts) every 2nd locker. Student 3 changes every 3rd locker, and so on, until student 100 has changed locker 100.

a. Which students change locker 12? Write down the factors of 12.
b. After all 100 students, is locker 12 open or shut? Explain using the number of factors.

c. Show that locker 36 is open at the end.
d. Which lockers are open at the end? How many are there?

Question 19

Question 19 – Higher: the product rule *(Section C · Problem solving)*

Higher tier. Use the product rule for counting: if there are a ways to do one thing and b ways to do another, there are $a \times b$ ways to do both.

a. A licence code has 3 different letters followed by 2 digits (0 to 9, repeats allowed). Letters can repeat. How many codes are there if repeats are allowed in the letters?
b. How many are there if the 3 letters must all be different?
c. A password has 4 digits using 0 to 9. How many passwords have no repeated digit?
d. How many of the passwords in (c) are even numbers (last digit even) and start with a non-zero digit?

Question 20

Question 20 – Challenge: numbers with many factors *(Section D · Challenge)*

The number of factors of $2^a \times 3^b \times 5^c$ is $(a+1)(b+1)(c+1)$.

a. Find the smallest whole number with exactly 12 factors.
b. Find the smallest whole number with exactly 10 factors.
c. Why must a number with exactly 3 factors be the square of a prime number?
d. Find the smallest number with exactly 3 factors that is greater than 50.