Answers - Factors, multiples, primes and HCF/LCM

Year: Year 10

Answer to Question 1

a. $1 \times 36,\ 2 \times 18,\ 3 \times 12,\ 4 \times 9,\ 6 \times 6$: the factors are **1, 2, 3, 4, 6, 9, 12, 18, 36**.
b. $1 \times 48,\ 2 \times 24,\ 3 \times 16,\ 4 \times 12,\ 6 \times 8$: the factors are **1, 2, 3, 4, 6, 8, 12, 16, 24, 48**.
c. The common factors are **1, 2, 3, 4, 6, 12**.
d. 36 has 9 factors and 48 has 10, so **48 has 1 more**.

Answer to Question 2

a. **23, 29, 31, 37, 41, 43, 47**
b. $91 = 7 \times 13$, so it has factors other than 1 and 91.
c. Every other even number is divisible by 2 as well as by 1 and itself, so it has at least three factors.
d. $3 + 37 = 40$, or $11 + 29 = 40$, or $17 + 23 = 40$. For example **3 and 37**.

Answer to Question 3

a. $180 = 2 \times 90 = 2 \times 2 \times 45 = 2^2 \times 3^2 \times 5$ Keep dividing by the smallest prime that goes in exactly: 2, then 3, then 5. So $180 = 2^2 \times 3^2 \times 5$.
b. $252 = 2 \times 126 = 2^2 \times 63 = 2^2 \times 3^2 \times 7$ Divide by 2 twice, then by 3 twice, which leaves the prime 7. So $252 = 2^2 \times 3^2 \times 7$.
c. $1176 = 2 \times 588 = 2^2 \times 294 = 2^3 \times 147 = 2^3 \times 3 \times 49 = 2^3 \times 3 \times 7^2$ Divide by 2 three times to reach 147, then by 3 to reach 49, which is 7×7. So $1176 = 2^3 \times 3 \times 7^2$.
d. $4 \times 9 \times 5 = 36 \times 5 =$ **180**. Multiplying the prime factors gives back the original number, so the answer to part (a) is correct. A factor tree gives the same primes in a different order.

Answer to Question 4

a. $24 = 2^3 \times 3$, $36 = 2^2 \times 3^2$. HCF $= 2^2 \times 3 =$ **12**. LCM $= 2^3 \times 3^2 =$ **72**.
b. $45 = 3^2 \times 5$, $75 = 3 \times 5^2$. HCF $= 3 \times 5 =$ **15**. LCM $= 3^2 \times 5^2 =$ **225**.
c. $28 = 2^2 \times 7$, $42 = 2 \times 3 \times 7$. HCF $= 2 \times 7 =$ **14**. LCM $= 2^2 \times 3 \times 7 =$ **84**.
d. $84 = 2^2 \times 3 \times 7$, $90 = 2 \times 3^2 \times 5$. HCF $= 2 \times 3 =$ **6**. LCM $= 2^2 \times 3^2 \times 5 \times 7 =$ **1260**. For the HCF take the lowest power of each common prime; for the LCM

take the highest power of every prime that appears.

Answer to Question 5

a. $576 = 2^6 \times 3^2$, so $\sqrt{576} = 2^3 \times 3 =$ **24**. The square root halves every power: 2^6 becomes 2^3 and 3^2 becomes 3. So $\sqrt{576} = 24$.
b. $1728 = 2^6 \times 3^3$, so $\sqrt[3]{1728} = 2^2 \times 3 =$ **12**. The cube root divides every power by 3: 2^6 becomes 2^2 and 3^3 becomes 3. So $\sqrt[3]{1728} = 12$.
c. $200 = 2^3 \times 5^2$. The power of 2 is odd, so it is not a perfect square. In a square number every prime appears to an even power, because it is a number times itself. So 200 is not a square number.
d. Multiply by 2: $400 = 2^4 \times 5^2 = 20^2$. So the answer is **2**. Multiplying by 2 makes the power of 2 even, so the smallest multiplier is 2.

Answer to Question 6

a. **6, 12, 18, 24, 30** The multiples of 6 are the numbers in the 6 times table.
b. **8, 16, 24, 32, 40** The multiples of 8 are the numbers in the 8 times table.
c. The first common multiple is **24** (LCM of 6 and 8). 24 is the first number that appears in both lists, so 24 is the smallest multiple of 6 and 8.
d. $6 = 2 \times 3$, $8 = 2^3$, $10 = 2 \times 5$: LCM $= 2^3 \times 3 \times 5 =$ **120**. Take the highest power of each prime: 2^3, 3 and 5. The smallest number divisible by 6, 8 and 10 is 120.

Answer to Question 7

a. The side must be a common factor of 420 and 300, so use the HCF: $420 = 2^2 \times 3 \times 5 \times 7$, $300 = 2^2 \times 3 \times 5^2$. HCF $= 60$. The largest tile is **60 cm**.
b. $420 \div 60 = 7$ and $300 \div 60 = 5$, so $7 \times 5 =$ **35 tiles**.
c. $35 \times 4.80 =$ **£168**.
d. The common factors of 420 and 300 are the factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60. Those at least 10: **10, 12, 15, 20, 30, 60 cm**.

Answer to Question 8

a. LCM of 12 and 18: $12 = 2^2 \times 3$, $18 = 2 \times 3^2$, LCM $= 2^2 \times 3^2 =$ **36 minutes**.
b. 08:00 plus 36 minutes is **08:36**.
c. Every 36 minutes in 240 minutes: $240 \div 36 = 6.67$, so there are 6 more times after 08:00, which is **7 times** including 08:00.
d. LCM of 12, 18 and 8: $2^3 \times 3^2 = 72$ minutes, so **09:12**.

Answer to Question 9

a. The number of bags divides both 84 and 126, so use the HCF: $84 = 2^2 \times 3 \times 7$, $126 = 2 \times 3^2 \times 7$, HCF $= 2 \times 3 \times 7 =$ **42 bags**.
b. $84 \div 42 =$ **2 chocolates** and $126 \div 42 =$ **3 sweets**.
c. Now there are 90 chocolates. 42 does not divide 90 exactly ($90 \div 42 = 2.14$), so he **cannot** make 42 bags with all the chocolates. The new HCF of 90 and 126 is 18.
d. The common factors of 84 and 126 are the factors of 42: **1, 2, 3, 6, 7, 14, 21, 42** bags.

Answer to Question 10

a. $10 = 2 \times 5$, $15 = 3 \times 5$, $25 = 5^2$. LCM $= 2 \times 3 \times 5^2 =$ **150 s**. For the LCM take the highest power of each prime: 2, 3 and 5^2. All three lights flash together again after 150 seconds.
b. 10 minutes $= 600$ s. $600 \div 150 =$ **4 times**. Change 10 minutes into seconds, then divide by 150. All three flash together 4 times.
c. $600 \div 10 =$ **60 flashes**. The red light flashes every 10 seconds, so it flashes 60 times in 600 seconds.
d. LCM of 10 and 15 = **30 s**. Using $10 = 2 \times 5$ and $15 = 3 \times 5$, the LCM is $2 \times 3 \times 5 = 30$. The red and green lights next flash together after 30 seconds.

Answer to Question 11

a. $360 = 2^3 \times 3^2 \times 5$.
b. Add 1 to each power and multiply: $(3+1)(2+1)(1+1) = 4 \times 3 \times 2 =$ **24 factors**.
c. The factors of 360 between 10 and 40 are **10, 12, 15, 18, 20, 24, 30, 36, 40**.
d. Rows $= 360 \div$ row length: **36, 30, 24, 20, 18, 15, 12, 10, 9** rows.

Answer to Question 12

a. LCM of 40 and 50: $40 = 2^3 \times 5$, $50 = 2 \times 5^2$, LCM $= 2^3 \times 5^2 =$ **200 s**.
b. Ali: $200 \div 40 =$ **5 laps**. Beth: $200 \div 50 =$ **4 laps**.
c. $600 \div 40 = 15$ laps for Ali and $600 \div 50 = 12$ laps for Beth.
d. LCM of 40 and 45 $= 2^3 \times 3^2 \times 5 =$ **360 s**. Here $45 = 3^2 \times 5$, so the LCM uses 2^3, 3^2 and 5. They next meet at the start after 360 seconds, which is 6 minutes.

Answer to Question 13

a. $4 \times 5 \times 3 =$ **60** Each choice of bread can go with each filling and each drink, so we multiply the numbers of choices: 4 breads, 5 fillings, 3 drinks. There are 60 different lunches.
b. $4 \times 4 \times 3 =$ **48** Without tuna there are only 4 fillings, so there are $4 \times 4 \times 3 = 48$ different lunches.
c. $4 \times 7 \times 3 =$ **84** With two new fillings there are $5 + 2 = 7$ fillings, so there are 84 different lunches.
d. $60 \div 5 =$ **12 weeks** There are 60 different lunches and she eats one each school day, so she can do it for 12 weeks.

Answer to Question 14

a. $\sqrt{144} =$ **12 rows** of 12. The number of rows equals the number of chairs in each row, so we take the square root. There are 12 rows of 12 chairs.
b. $13 \times 13 =$ **169 chairs** $(144 + 12 + 12 + 1)$. The new square is 13 by 13. There are now 169 chairs.
c. $\sqrt{2025} =$ **45** per row $(45^2 = 2025)$. Use the square root key or prime factors: $2025 = 3^4 \times 5^2$, so $\sqrt{2025} = 3^2 \times 5 = 45$. There are 45 chairs in each row.
d. $14^2 = 196 < 200 < 225 = 15^2$, and $200 = 2^3 \times 5^2$ has an odd power of 2, so it is not square. Another $225 - 200 =$ **25 chairs** are needed. So 200 chairs cannot make a square, and 25 more chairs are needed to make 225.

Answer to Question 15

a. $(2a + 1) + (2b + 1) = 2a + 2b + 2 = 2(a + b + 1)$, which is 2 times a whole number, so it is **even**.
b. $(2a + 1)(2b + 1) = 4ab + 2a + 2b + 1 = 2(2ab + a + b) + 1$, which is 2 times a whole number plus 1, so it is **odd**.
c. $n = 3$: $9 + 3 = 12$, even. $n = 4$: $16 + 4 = 20$, even. In general $n^2 + n = n(n + 1)$, a product of two consecutive numbers. One of them is even, so the product is **always even**.
d. $(2a + 1) + 2b = 2(a + b) + 1$, which is 2 times a whole number plus 1, so **yes, it is always odd**.

Answer to Question 16

a. HCF $= 2^2 \times 3^2 = 2^2 \times 3^2$ **(that is 36)**. For the HCF take the lowest power of each prime that is in both numbers. The HCF is $2^2 \times 3^2 = 36$.
b. LCM $= 2^3 \times 3^3 \times 5 \times 7 = 2^3 \times 3^3 \times 5 \times 7$ **(that is 7560)**. For the LCM take the highest power of every prime that appears. The LCM is 7560.
c. $A \times B = 2^5 \times 3^5 \times 5 \times 7$. HCF $\times$ LCM

$= 2^2 \times 3^2 \times 2^3 \times 3^3 \times 5 \times 7 = 2^5 \times 3^5 \times 5 \times 7$, so they are equal. When multiplying, add the indices of the same prime.

d. $A = 8 \times 9 \times 5 = 360$, $B = 4 \times 27 \times 7 = 756$, HCF $= 36$ (and LCM $= 7560$). Check: $360 \times 756 = 272\,160$ and $36 \times 7560 = 272\,160$.

Answer to Question 17

a. The product of the two numbers is HCF $\times$ LCM $= 6 \times 72 = 432$. So the other number is $432 \div 18 =$ **24**.

b. $18 = 2 \times 3^2$, $24 = 2^3 \times 3$. HCF $= 2 \times 3 = 6$. LCM $= 2^3 \times 3^2 = 72$. Both are correct.

c. The numbers are multiples of 6 whose product is 432. Try $6k$ and $6m$ with $k \times m = 12$ and k, m having no common factor: $(k, m) = (3, 4)$ gives $18, 24$. Also $(1, 12)$ gives $6, 72$ but 6 is not bigger than 6. So the only pair is **18 and 24**.

d. LCM of 8, 12 and 15 is $2^3 \times 3 \times 5 = 120$. To get a square, each prime needs an even power: multiply by $2 \times 3 \times 5 = 30$ to get $3600 = 60^2$. The answer is **3600**.

Answer to Question 18

a. Student k changes locker 12 if k is a factor of 12: students **1, 2, 3, 4, 6, 12**.

b. Locker 12 is changed 6 times. It starts shut, so an even number of changes leaves it **shut**.

c. The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36: that is 9, an odd number of changes, so locker 36 is **open**.

d. A locker is open only if its number has an odd number of factors, which happens only for square numbers (factors come in pairs except the square root). The open lockers are **1, 4, 9, 16, 25, 36, 49, 64, 81, 100**, which is **10 lockers**.

Answer to Question 19

a. $26 \times 26 \times 26 \times 10 \times 10 =$ **1 757 600**

b. $26 \times 25 \times 24 \times 10 \times 10 =$ **1 560 000**

c. $10 \times 9 \times 8 \times 7 =$ **5040**

d. Last digit even: 5 choices (0, 2, 4, 6, 8). First digit non-zero and different from the last digit. Case last digit 0: first digit 9 choices, then 8×7: $9 \times 8 \times 7 = 504$. Case last digit 2, 4, 6 or 8 (4 choices): first digit 8 choices (non-zero, not the last), then 8×7 for the middle two: $4 \times 8 \times 8 \times 7 = 1792$. Total $= 504 + 1792 =$ **2296**.

Answer to Question 20

a. $12 = 12$, 6×2, 4×3 or $3 \times 2 \times 2$. Try $2^5 \times 3 = 96$, $2^3 \times 3^2 = 72$, $2^2 \times 3 \times 5 = 60$ and $2^{11} = 2048$. The smallest is **60**.

b. $10 = 10$ or 5×2. Try $2^9 = 512$ and $2^4 \times 3 = 48$. The smallest is **48**.

c. If $N = p^2$ for a prime p, its factors are 1, p, p^2, which is three. If N had two different prime factors it would have at least four factors, so only p^2 works.
d. The squares of primes are $4, 9, 25, 49, 121$. The smallest above 50 is **121** (11^2).