Lesson - Compound units and direct/inverse proportion

Year: Year 10

Compound units combine two measurements, such as speed (distance per time), density (mass per volume) and pressure (force per area). Direct and inverse proportion describe how one quantity changes as another changes. On the GCSE these topics are on both tiers, but constructing equations for inverse proportion and proportion to powers (such as $y \propto x^2$) are Higher tier only.

Starter activity

A school minibus leaves at 8:00 and reaches a museum 54 km away at 9:15. Work out how long the trip took in hours. Estimate its average speed in km/h. Then ask: if it had travelled at 60 km/h for the same distance, how many minutes quicker would it have been? Next, if a second bus is twice as fast, how long would it take for the same journey? What does this tell you about speed and time when the distance is fixed?

Speed, distance and time

Definition: compound unit

A **compound unit** combines two measurements, such as km/h (distance per time), g/cm^3 (mass per volume) or N/m^2 (force per area).

Rule: speed, distance and time

The formula triangle for speed is: $\text{speed} = \dfrac{\text{distance}}{\text{time}}$, $\text{distance} = \text{speed} \times \text{time}$, $\text{time} = \dfrac{\text{distance}}{\text{speed}}$. The units must match: km and hours give km/h; m and seconds give m/s.

- 150 km in 2.5 h: speed $= \frac{150}{2.5} = 60$ km/h.
- 45 mph for 2 h 20 min: $2\text{ h }20\text{ min} = 2\frac{1}{3}$ h, distance $= 45 \times \frac{7}{3} = 105$ miles. Convert minutes to a fraction of an hour by dividing by 60 (never write 2.2 hours).
- 180 km at 72 km/h: time $= \frac{180}{72} = 2.5$ h $= 2$ h 30 min.
- Convert km/h to m/s: $\text{km/h} \times \frac{1000}{3600}$. 90 km/h $= 25$ m/s.

Average speed is the *total distance divided by the total time*, not the average of two speeds. 60 km at 40 km/h takes 1.5 h; 60 km at 60 km/h takes 1 h. Average $= \frac{120}{2.5} = 48$ km/h.

Density

Rule: density

$\text{density} = \dfrac{\text{mass}}{\text{volume}}$, with units such as g/cm^3 or kg/m^3. A block of 540 g and volume 200 cm^3 has density 2.7 g/cm^3. Mass = density $\times$ volume: $8.9 \times 25 = 222.5$ g. Volume = mass $\div$ density: $360 \div 0.9 = 400$ cm^3. For a mixture, add the masses and add the volumes first. 100 cm^3 of metal at 8 g/cm^3 plus 150 cm^3 at 4 g/cm^3 has mass $800 + 600 = 1400$ g and volume 250 cm^3, so density 5.6 g/cm^3.

Pressure

Rule: pressure

$\text{pressure} = \dfrac{\text{force}}{\text{area}}$. The unit is N/m^2 (pascals, Pa) when force is in newtons and area in m^2. 600 N on 0.2 m^2 gives 3000 N/m^2. Force = pressure $\times$ area: 2500 Pa on 0.4 m^2 is 1000 N. The same force on a smaller area gives a larger pressure, which is why knives are sharp.

Direct proportion

Definition: direct proportion

y is **directly proportional** to x if $y = kx$. The graph is a straight line through the origin.

Rule: direct proportion

y is directly proportional to x ($y \propto x$) if $y = kx$ for a constant k. The graph is a straight line through the origin. If $y = 15$ when $x = 6$, then $k = 2.5$, $y = 2.5x$, and $y = 40$ when $x = 16$. If 3 m of fabric costs £13.50 then 8 m costs $\frac{13.5}{3} \times 8 = £36$.

Inverse proportion

Definition: inverse proportion

y is **inversely proportional** to x if $y = \dfrac{k}{x}$. When one quantity doubles, the other halves.

Rule: inverse proportion

y is inversely proportional to x if $y = \dfrac{k}{x}$: when one doubles the other halves. If 6 workers take 10 days, the total work is 60 worker-days, so 4 workers take $\frac{60}{4} = 15$ days. If 3 pumps take 8 hours, 12 pumps take 2 hours. The graph is a curve that never touches the axes.

Constructing equations (Higher tier)

Method: constructing equations (Higher tier)

Write $y = \dfrac{k}{x}$, substitute the known pair to find k, then use the equation. If $y = 6$ when $x = 4$: $k = 24$, so $y = \frac{24}{x}$ and $y = 3$ when $x = 8$. For $y \propto x^2$: $y = 50$ when $x = 5$ gives $k = 2$, $y = 2x^2$, and when $x = 8$, $y = 128$. If y is inversely proportional to the square of x and $y = 4$ when $x = 3$, then $k = 36$; when $x = 6$, $y = \frac{36}{36} = 1$. For travel, $t = \dfrac{k}{v}$: 2 h at 30 mph gives $k = 60$, so at 45 mph the time is $\frac{60}{45} = 1\frac{1}{3}$ h $= 1$ h 20 min.

More worked examples in exam style

Worked example 1

A runner takes 25 minutes to run 5 km. Find her speed in km/h [3 marks].

25 min $= \frac{25}{60}$ h. Speed $= 5 \div \frac{25}{60} = 12$ km/h.

Worked example 2

The density of oak is 0.75 g/cm^3. A plank has volume 4000 cm^3. Find its mass in kg [3 marks].

Mass $= 0.75 \times 4000 = 3000$ g $= 3$ kg.

Worked example 3

The cost C in pounds of hiring a hall for h hours is directly proportional to h. Hiring for 5 hours costs £120. Find the cost of 8 hours [3 marks].

$C = 24h$. $24 \times 8 = £192$.

Worked example 4 (Higher tier)

The pressure P of a gas is inversely proportional to its volume V. $P = 200$ when $V = 3$. Find P when $V = 5$ [3 marks].

$P = \frac{k}{V}$, $k = 600$. $P = \frac{600}{5} = 120$.

Common mistakes

Watch out

- Dividing in the wrong order: use the triangle and check the units.
- Writing 2 h 20 min as 2.2 hours instead of $2\frac{1}{3}$.
- Finding average speed by adding speeds and halving.
- Mixing units (g with kg, cm^3 with m^3).
- Treating inverse proportion as "subtract" instead of using $y = \frac{k}{x}$.

Exam technique

Exam technique

- Write the formula triangle and cover the quantity you want.
- Convert minutes to hours before dividing, and state the units in your answer.
- For proportion questions, find the constant k first and write the equation.

Compound units and the formula triangle

A compound unit combines two other units. Speed is distance per time (km/h or m/s), density is mass per volume (g/cm³ or kg/m³) and pressure is force per area (N/m² or N/cm²). Each quantity has three versions of the same formula, and a formula triangle helps you pick the right one. Put the quantity that is a division on top and the two multiplied quantities along the bottom. For speed: D on top and S, T below, so $S = D \div T$, $D = S \times T$ and $T = D \div S$. Always check that the units match before you substitute. If the speed is in km/h and the time is in minutes, convert the time to hours first.

Worked example 5: time in hours and minutes

A train travels 245 km at an average speed of 98 km/h. Find the time of the journey in hours and minutes [3 marks].

Time $= \text{distance} \div \text{speed} = 245 \div 98 = 2.5$ hours. The decimal part 0.5 hours is $0.5 \times 60 = 30$ minutes, so the time is 2 hours 30 minutes. A common slip is to write 2.5 hours as 2 hours 50 minutes. Always convert a decimal part of an hour by multiplying by 60.

Worked example 6: unit conversion for density

A steel ball has volume 65 cm³ and mass 0.5 kg. Find its density in g/cm³ and in kg/m³ [3 marks].

Change the mass to grams: $0.5 \times 1000 = 500$ g. Density $= 500 \div 65 = 7.69$ g/cm³ to 3 significant figures. To change to kg/m³, multiply by 1000: $7.69 \times 1000 = 7690$ kg/m³. Check: steel is known to be about 7800 kg/m³, so the answer is sensible.

Worked example 7: setting up inverse proportion

It takes 5 painters 8 days to paint a school. How long would 4 painters take, and how many painters are needed to finish in 5 days? [4 marks]

The product $\text{painters} \times \text{days}$ is constant. $5 \times 8 = 40$ painter-days. With 4 painters: $40 \div 4 = 10$ days. To finish in 5 days: $40 \div 5 = 8$ painters. Fewer painters means more days, and more painters means fewer days, which is how inverse proportion behaves.

Worked example 8: an unusual pressure question

A cylinder-shaped can exerts a force of 12 N on the table. The base has radius 3 cm. Find the pressure in N/cm² to 2 decimal places [3 marks].

The base is a circle, so area $= \pi \times 3^2 = 28.27$ cm². Pressure $= 12 \div 28.27 = 0.42$ N/cm². Notice that the area is found first, using the circle area formula, before the pressure formula is used.

Note: recognising proportion

Two quantities are in direct proportion if they increase together and their ratio stays constant, and their graph is a straight line through the origin. They are in inverse proportion if one goes up when the other goes down and the product stays constant, and their graph is a curve. Test a table by dividing for direct proportion and multiplying for inverse proportion. A cost graph with a fixed starting fee is a straight line, but not through the origin, so it is not direct proportion.

Watch out: average speed

The average speed for a whole journey is always the *total distance divided by the total time*. It is not the mean of two speeds unless the times are equal. For a return journey over the same distance at two different speeds, the average is always below the mean of the speeds.

Did you know?

The unit of force, the newton, is named after Sir Isaac Newton, and the unit of pressure, the pascal (Pa, equal to 1 N/m²), is named after the French mathematician and physicist Blaise Pascal. A pressure of one pascal is tiny, about the pressure of a banknote lying flat on a table, so weather forecasters use hectopascals, and normal air pressure at sea level is about 1013 hectopascals. The idea of density goes back to Archimedes, who is said to have shouted “Eureka!” when he realised that he could tell whether a crown was pure gold by measuring the water it displaced and comparing its density with that of gold.

Otto's tip

Write the units with every number. If your answer is in the wrong units, or your speed is faster than a plane, something went wrong. For proportion, always find k first.