Answers - Compound units and direct/inverse proportion

Year: Year 10

Answer to Question 1

a. $150 \div 2.5 =$ **60 mph** Divide the distance by the time. The coach's average speed is 60 mph.
b. 2 hours 20 minutes $= 2\frac{1}{3}$ hours $= 2.333$ h. Distance $= 18 \times 2.333 =$ **42 km** Change the minutes into a fraction of an hour first: 20 minutes is $\frac{20}{60} = \frac{1}{3}$ of an hour. Then multiply speed by time: she rides 42 km.
c. $240 \div 80 =$ **3 hours** Rearrange the formula: time = distance ÷ speed. The journey takes 3 hours.
d. $72\ \text{km/h} = 72\,000\ \text{m per hour} = 72\,000 \div 3600 =$ **20 m/s** Change kilometres into metres ($\times 1000$) and hours into seconds ($\times 3600$). So 72 km/h is 20 m/s.

Answer to Question 2

a. $540 \div 200 =$ **2.7 g/cm³** Divide the mass by the volume. The density of the block is 2.7 g/cm³.
b. $0.8 \times 250 =$ **200 g** Rearrange the formula: mass = density × volume. The mass of the liquid is 200 g.
c. $1.2\ \text{kg} = 1200\ \text{g}$, so volume $= 1200 \div 8 =$ **150 cm³** The units must match, so change kilograms into grams before dividing. The volume is 150 cm³.
d. $2.7\ \text{g/cm}^3 = 2700\ \text{kg/m}^3$ (multiply by 1000). 1 g/cm³ is the same as 1000 kg/m³, because 1 m³ holds 1 000 000 cm³ and 1 kg is 1000 g. So the density is 2700 kg/m³.

Answer to Question 3

a. $600 \div 0.4 =$ **1500 N/m²** Divide the force by the area. The pressure is 1500 N/m².
b. $25 \times 12 =$ **300 N** Rearrange the formula: force = pressure × area. The force is 300 N.
c. $840 \div 3500 =$ **0.24 m²** Rearrange the formula: area = force ÷ pressure. The area is 0.24 m².
d. $90 \div 15 =$ **6 N/cm²** The area is in cm², so the answer is in N/cm². The pressure is 6 N/cm².

Answer to Question 4

a. 1 pen $= 3.45 \div 5 = £0.69$. 8 pens $= 0.69 \times 8 =$ **£5.52** First find the cost of one pen, then multiply by 8. So 8 pens cost £5.52.
b. 1 person $= 280 \div 4 = 70$ g. 10 people $=$ **700 g** First find the amount for one person, then multiply by 10. So 700 g of pasta is needed.
c. 1 litre $= 75 \div 6 = 12.5$ km. 10 litres $=$ **125 km** Find the distance for 1 litre, then multiply by 10. The car can go 125 km.
d. 1 notebook $= 9.45 \div 7 = £1.35$. $20 \div 1.35 = 14.8$, so **14 notebooks** costing $14 \times 1.35 = 18.90$, leaving **£1.10** change. We round down because we cannot buy part of a notebook. So 14 notebooks can be bought with £1.10 change.

Answer to Question 5

a. Total work $= 6 \times 10 = 60$ worker-days. $60 \div 4 =$ **15 days**
b. $8 \times 9 = 72$ pump-hours. $72 \div 12 =$ **6 hours**
c. $20 \times 15 = 300$ bird-days. $300 \div 25 =$ **12 days**
d. Fewer workers take longer, not less time: 3 workers take **twice** as long (20 days), because the time is inversely proportional to the number of workers.

Answer to Question 6

a. $5 \times 3600 = 18\,000$ m per hour $=$ **18 km/h** There are 3600 seconds in an hour, and 1000 metres in a kilometre. So 5 m/s is 18 km/h.
b. $1.5 \times 1000 =$ **1500 kg/m³** 1 g/cm³ equals 1000 kg/m³, so multiply by 1000. The density is 1500 kg/m³.
c. $36 \times 1000 \div 3600 =$ **10 m/s** Change kilometres into metres, then divide by the number of seconds in an hour. So 36 km/h is 10 m/s.
d. $1\ \text{m}^2 = 10\,000\ \text{cm}^2$, so $250 \times 10\,000 =$ **2 500 000 N/m²** Each square metre contains 10 000 square centimetres, so the force on 1 m² is 10 000 times as big. The pressure is 2 500 000 N/m².

Answer to Question 7

a. First: $30 \div 1.5 =$ **20 km/h**. Second: $20 \div 0.5 =$ **40 km/h**.
b. Total distance $= 50$ km. Total time $= 2$ h. Average speed $= 50 \div 2 =$ **25 km/h**.
c. The mean of 20 and 40 is 30 km/h, but the true average is 25 km/h. More time was spent travelling at the slower speed, so the average speed must be nearer 20.
d. $75 \div 3 =$ **25 km/h**.

Answer to Question 8

a. 09:15 to 11:45 is 2 hours 30 minutes $=$ **2.5 hours**. Count on from 09:15: 2 hours takes us to 11:15, then 30 more minutes to 11:45.
b. $190 \div 2.5 =$ **76 mph**. Divide the distance by the time in hours. The average speed is 76 mph.
c. 3 h 10 min $= 3\frac{1}{6} = 3.1667$ h. $190 \div 3.1667 =$ **60 mph**. 10 minutes is $\frac{10}{60} = \frac{1}{6}$ of an hour. The slower train's average speed is about 60 mph.
d. 190 min $- 150$ min $=$ **40 minutes** quicker. The faster train takes 150 minutes and the slower one takes 190 minutes. So the faster train is 40 minutes quicker.

Answer to Question 9

a. $10 \times 5 \times 2 =$ **100 cm³**. The bar is a cuboid, so we multiply length, width and height. The volume is 100 cm³.
b. $19.3 \times 100 =$ **1930 g** $=$ **1.93 kg**. Use mass $=$ density $\times$ volume, then divide by 1000 to change grams into kilograms. The bar has a mass of 1930 g, which is 1.93 kg.
c. $1930 \times 50 =$ **£96 500**. Multiply the mass in grams by the price per gram. The bar is worth £96 500.
d. $800 \div 100 = 8$ g/cm³. This is much less than 19.3 g/cm³, so the bar is **not pure gold**. The density of a substance does not depend on the size of the object, so a pure gold bar of this size would weigh 1930 g.

Answer to Question 10

a. Ashby: $54\,000 \div 36 =$ **1500/km²**. Barnfield: $12\,600 \div 90 =$ **140/km²**. Corton: $880\,000 \div 400 =$ **2200/km²**. Divide each population by the area in km².
b. **Corton** has the highest density. The most crowded place has the most people per square kilometre, so Corton is the most crowded with 2200 people/km².
c. $1200 \times 25 =$ **30 000 people**. Multiply the density by the area. About 30 000 people will live in the new town.
d. New population $12\,600 \times 1.2 = 15\,120$. Density $= 15\,120 \div 90 =$ **168/km²**. A 20% increase means multiplying by 1.2. The new density of Barnfield is 168 people/km².

Answer to Question 11

a. $600 \div 400 =$ **1.5 N/cm²** Divide the force (her weight) by the area in contact with the floor. The pressure in trainers is 1.5 N/cm².
b. $600 \div 1 =$ **600 N/cm²** The whole weight presses on 1 cm². The pressure under the heel is 600 N/cm².
c. $600 \div 1.5 =$ **400 times** bigger. Divide the larger pressure by the smaller one. The pressure under the heel is 400 times bigger.
d. The same force acting on a much smaller area gives a much bigger pressure, so

the heel can dent the floor. Her weight is the same in both cases, but the area under the heel is 400 times smaller.

Answer to Question 12

a. Scale factor $20 \div 8 = 2.5$: butter **300 g**, sugar **225 g**, flour **450 g**, eggs **2.5**.
b. 1 cookie uses $120 \div 8 = 15$ g butter. $250 \div 15 = 16.7$, so **16 cookies**.
c. 1 cookie uses $180 \div 8 = 22.5$ g flour. $500 \div 22.5 = 22.2$, so **22 cookies**.
d. 3 eggs make $3 \times 8 = 24$ cookies, which is more than 22, so the eggs do not limit her; the flour does.

Answer to Question 13

a. $450 \div 100 \times 6.4 =$ **28.8 litres** The journey is 4.5 lots of 100 km, so multiply 6.4 litres by 4.5. The family needs 28.8 litres of petrol.
b. $28.8 \times 1.52 = 43.776$, so **£43.78** Multiply the litres by the price per litre and round to the nearest penny. The petrol costs £43.78.
c. Driving time $= 5.5 - 0.5 = 5$ hours. Speed $= 450 \div 5 =$ **90 km/h**. The half-hour break is not driving time, so subtract it before dividing. The average speed while driving is 90.0 km/h.
d. $43.776 \div 450 = 0.0973$ £/km $=$ **9.7p per km**. Divide the total cost by the distance, then multiply by 100 to change pounds into pence. The cost is 9.7p per kilometre.

Answer to Question 14

a. Total work $= 4 \times 6 = 24$ person-hours. $24 \div 3 =$ **8 hours**.
b. $24 \div 8 =$ **3 hours**. Twice as many people take half the time, so 8 people take 3 hours.
c. 4 people: $4 \times 6 \times 12 =$ **£288**. 8 people: $8 \times 3 \times 12 =$ **£288**.
d. The wage bills are the same because the total number of person-hours (24) is the same however many people share the job.

Answer to Question 15

a. Out: $36 \div 12 = 3$ h. Back: $36 \div 18 = 2$ h.
b. Total distance $= 72$ km, total time $= 5$ h, so average speed $= 72 \div 5 =$ **14.4 km/h**.
c. She spends more time at the slower speed, so the true average (14.4) is below the mean (15).
d. Total time for 16 km/h $= 72 \div 16 = 4.5$ h. Time back $= 4.5 - 3 = 1.5$ h. Speed back $= 36 \div 1.5 =$ **24 km/h**.

Answer to Question 16

a. $300 \div 8.9 =$ **33.71 cm³** Volume = mass ÷ density. The copper has a volume of 33.71 cm³.
b. $100 \div 7.3 =$ **13.70 cm³** Use the same rearrangement for the tin. Its volume is 13.70 cm³.
c. Mass $= 300 + 100 = 400$ g. Volume $= 33.71 + 13.70 =$ **47.41 cm³**. The masses and volumes of the two metals are added together.
d. $400 \div 47.41 =$ **8.44 g/cm³**. It lies between 7.3 and 8.9 because the alloy is a mixture of the two metals. Density of the alloy = total mass ÷ total volume. There is more copper than tin, so the density is closer to 8.9.

Answer to Question 17

a. $45 \times 1.609 =$ **72.4 km** per gallon. One gallon takes the car 45 miles, and each mile is 1.609 km. So the car goes about 72.4 km on one gallon.
b. $72.4 \div 4.546 =$ **15.9 km/litre** (15.93). One gallon is 4.546 litres, so divide the distance by 4.546. The car does about 15.9 km per litre.
c. $100 \div 15.93 =$ **6.3 litres per 100 km**. To go 100 km, divide 100 by the distance for one litre. The car uses about 6.3 litres per 100 km.
d. $600 \div 15.93 = 37.7$ litres. Cost $= 37.7 \times 1.50 =$ **£56.50** (about). Divide 600 km by the distance per litre, then multiply by the price. The trip uses about 37.7 litres, which costs about £56.50.

Answer to Question 18

a. $2 = k\sqrt{1}$, so $k = 2$ and $T = 2\sqrt{L}$. Substitute the known pair of values, $L = 1$ and $T = 2$, to find the constant. The formula is $T = 2\sqrt{L}$.
b. $T = 2\sqrt{0.64} = 2 \times 0.8 =$ **1.6 s**. Substitute $L = 0.64$ into the formula; $\sqrt{0.64} = 0.8$. One swing takes 1.6 seconds.
c. $5 = 2\sqrt{L}$, so $\sqrt{L} = 2.5$ and $L =$ **6.25 m**. Divide by 2, then square both sides to undo the square root. The pendulum is 6.25 m long.
d. $T = 2\sqrt{4L} = 2 \times 2\sqrt{L} = 2T$, so T **doubles**. Because $\sqrt{4} = 2$, multiplying the length by 4 multiplies the time by 2.

Answer to Question 19

a. $36 = \frac{k}{4}$, so $k =$ **144**. Substitute $d = 2$ and $I = 36$ into the formula, then multiply both sides by 4. So $k = 144$ and $I = \frac{144}{d^2}$.

b. $I = \dfrac{144}{9} =$ **16 units**. Substitute $d = 3$, so $d^2 = 9$. The brightness at 3 m is 16 units.
c. $9 = \dfrac{144}{d^2}$, $d^2 = 16$, $d =$ **4 m**. Rearrange to $d^2 = 144 \div 9$, then take the positive square root because a distance is positive. The brightness is 9 units at 4 m.
d. If d is doubled, d^2 is multiplied by 4, so the brightness is divided by 4: it falls to **a quarter**. For example, from 2 m to 4 m the brightness falls from 36 to 9 units, which is a quarter.

Answer to Question 20

a. $1 \div 30 = \frac{1}{30}$ h = **2 minutes**.
b. $2 \div 60 = \frac{1}{30}$ h = **2 minutes**.
c. The whole journey should take only 2 minutes, but he has already used 2 minutes on the first mile, leaving **no time** for the second mile. It would need infinite speed.
d. Second mile: $1 \div 90 = \frac{1}{90}$ h $= 0.667$ min. Total time $= 2.667$ min. Average speed $= 2 \div (2.667/60) =$ **45 mph**.