Answers - Area, volume, surface area, arcs and sectors

Year: Year 10

Answer to Question 1

a. $9 \times 6 =$ **54 cm²** The area of a parallelogram is base times perpendicular height, not the slanted side. The area is 54 cm².
b. $\frac{1}{2} \times 14 \times 8 =$ **56 cm²** A triangle is half of a parallelogram with the same base and height, so we halve $14 \times 8 = 112$. The area is 56 cm².
c. $\frac{1}{2}(7 + 13) \times 5 = \frac{1}{2} \times 20 \times 5 =$ **50 cm²** Add the parallel sides, halve the total and multiply by the distance between them. The area of the trapezium is 50 cm².
d. $\pi \times 6^2 = 36\pi =$ **113.1 cm²** Square the radius first, then multiply by π: $36\pi = 113.097\ldots$, which rounds to 113.1 cm² to 1 decimal place.

Answer to Question 2

a. $8 \times 5 - 3 \times 2 = 40 - 6 =$ **34 m²**
b. The perimeter of an L-shape cut from a corner equals the perimeter of the full rectangle: $2(8 + 5) =$ **26 m**.
c. $34 \times 14.50 =$ **£493**
d. Cutting a corner out moves two sides inwards by the same lengths that the other two sides are shortened, so the total length of the boundary does not change.

Answer to Question 3

a. $12 \times 5 \times 4 =$ **240 cm³** Multiply the three dimensions together. The volume of the cuboid is 240 cm³.
b. volume = area of cross-section $\times$ length $= 24 \times 10 =$ **240 cm³** Any prism has the same cross-section all the way along, so the volume is 240 cm³.
c. $\pi \times 4^2 \times 10 = 160\pi =$ **502.7 cm³** A cylinder is a prism with a circular cross-section of area $\pi r^2 = 16\pi$. The volume is 502.7 cm³ to 1 decimal place.
d. radius = 5 cm, so $\pi \times 25 \times 6 = 150\pi =$ **471.2 cm³** Halve the diameter before using the formula: a common mistake is to use 10 as the radius. The volume is 471.2 cm³.

Answer to Question 4

a. $2(8 \times 5 + 8 \times 3 + 5 \times 3) = 2(40 + 24 + 15) =$ **158 cm²** A cuboid has three pairs of identical faces, so we find one face of each pair and double the total. The surface

area is 158 cm².

b. $2\pi r^2 + 2\pi rh = 2\pi \times 9 + 2\pi \times 3 \times 8 = 18\pi + 48\pi = 66\pi =$ **207.3 cm²** The two circular ends give $2\pi r^2$ and the curved surface opens out into a rectangle of length $2\pi r$ and height h. The surface area is 207.3 cm².

c. $6 \times 49 =$ **294 cm²** A cube has 6 identical square faces, each of area $7^2 = 49$ cm². The surface area is 294 cm².

d. Two triangles: $2 \times \frac{1}{2} \times 3 \times 4 = 12$. Three rectangles: $10 \times (3 + 4 + 5) = 120$. Total $=$ **132 cm²**. The 3-4-5 triangle has a right angle between the 3 cm and 4 cm sides, so its area is $\frac{1}{2} \times 3 \times 4$. The total surface area of the prism is 132 cm².

Answer to Question 5

a. $2.5 \times 10\,000 =$ **25 000 cm²** One square metre is $100\text{ cm} \times 100\text{ cm}$, so we multiply by 10 000, not 100. So $2.5\text{ m}^2 = 25\,000\text{ cm}^2$.

b. $3 \times 100 =$ **300 mm²** One square centimetre is $10\text{ mm} \times 10\text{ mm}$, so we multiply by 100. So $3\text{ cm}^2 = 300\text{ mm}^2$.

c. $4500 \div 1000 =$ **4.5 litres** There are 1000 cm³ in one litre, so we divide by 1000. So $4500\text{ cm}^3 = 4.5$ litres.

d. $0.6\text{ m}^3 = 600\,000\text{ cm}^3 =$ **600 litres** First multiply by 1 000 000 to change m³ into cm³, then divide by 1000 to change into litres. In one step, $1\text{ m}^3 = 1000$ litres, so $0.6\text{ m}^3 = 600$ litres.

Answer to Question 6

a. $\frac{75}{360} \times 2\pi \times 12 = 5\pi =$ **15.7 cm** The fraction $\frac{75}{360}$ tells us what part of the full circle the sector is. The arc length is 15.7 cm.

b. $\frac{75}{360} \times \pi \times 144 = 30\pi =$ **94.2 cm²** Use the same fraction of the full circle's area $\pi \times 12^2 = 144\pi$. The sector area is 94.2 cm².

c. $\frac{144}{360} \times 2\pi \times 10 = 8\pi =$ **25.1 cm** Here $\frac{144}{360} = 0.4$, so the arc is 40% of the circumference 20π. The arc length is 25.1 cm.

d. $\frac{144}{360} \times \pi \times 100 = 40\pi =$ **125.7 cm²** The sector is 40% of the circle's area 100π, so the area is 125.7 cm².

Answer to Question 7

a. $25 \times 10 \times 1.5 =$ **375 m³**, and $375 \times 1000 =$ **375 000 litres**. There are 1000 litres in 1 m³. The pool holds 375 m³, which is 375 000 litres.

b. $375\,000 \div 500 = 750$ minutes $=$ **12.5 hours**. Divide the minutes by 60 to change to hours: $750 \div 60 = 12.5$. The pool takes 12.5 hours to fill.

c. Floor: $25 \times 10 = 250$. Long walls: $2 \times 25 \times 1.5 = 75$. Short walls: $2 \times 10 \times 1.5 = 30$. Total $= 250 + 75 + 30 =$ **355 m²**. The top is open water, so only five faces are tiled. The area to be tiled is 355 m².
d. $355 \times 18 =$ **£6390**. Multiply the area by the price per square metre. The tiles cost £6390.

Answer to Question 8

a. $\pi \times 3.5^2 \times 10 = 122.5\pi = 384.8$, so **385 cm³**.
b. $2\pi \times 3.5 \times 10 = 70\pi =$ **219.9 cm²**.
c. Each can is 7 cm across. Length $= 4 \times 7 = 28$ cm, width $= 3 \times 7 = 21$ cm, height $= 10$ cm. Volume $= 28 \times 21 \times 10 =$ **5880 cm³**.
d. Cans: $12 \times 122.5\pi = 1470\pi = 4618.1$ cm³. Empty space $= 5880 - 4618.1 =$ **1261.9 cm³** (about 1262 cm³).

Answer to Question 9

a. $360 \div 8 =$ **45°** The 8 slices are equal, so the full turn of 360° is shared equally between them. Each slice has an angle of 45°.
b. radius 15 cm: $\frac{45}{360} \times \pi \times 225 = 28.125\pi =$ **88.4 cm²** The radius is half of 30 cm, and one slice is $\frac{1}{8}$ of the whole pizza. The area of one slice is 88.4 cm².
c. $\frac{45}{360} \times 2\pi \times 15 = 3.75\pi =$ **11.8 cm** The crust is the arc of the sector, which is $\frac{1}{8}$ of the circumference 30π. The crust of one slice is 11.8 cm long.
d. Box: $32^2 = 1024$. Pizza: $225\pi = 706.9$. Uncovered: $1024 - 706.9 =$ **317.1 cm²**. Subtract the area of the circle from the area of the square base. The uncovered area is 317.1 cm².

Answer to Question 10

a. Rectangle $3 \times 2.2 = 6.6$. Triangle $\frac{1}{2} \times 3 \times 0.8 = 1.2$. Total $=$ **7.8 m²**.
b. $7.8 \times 2.5 =$ **19.5 m³**. The volume of a prism is the area of the cross-section times its length, so the volume of the shed is 19.5 m³.
c. Half the width is 1.5 m, so slope $= \sqrt{1.5^2 + 0.8^2} = \sqrt{2.89} =$ **1.7 m**. The slope is the hypotenuse of a right-angled triangle with sides 1.5 m and 0.8 m. The slope length is 1.7 m.
d. Roof area $= 2 \times 1.7 \times 2.5 = 8.5$ m². Cost $= 8.5 \times 4.20 =$ **£35.70**. There are two panels, each 1.7 m by 2.5 m. The felt costs £35.70.

Answer to Question 11

a. $\pi \times 0.36 \times 1.4 = 0.504\pi = 1.583$ m³ = **1583 litres**. The volume of a cylinder is $\pi r^2 h$ and 1 m³ is 1000 litres. The water butt holds 1.583 m³, which is 1583 litres.
b. $1583 \div 20 = 79.2$, so **79 minutes**. Divide the volume in litres by the rate. It takes about 79 minutes to fill.
c. $0.9\ \text{m}^3 \div (\pi \times 0.36) = 0.9 \div 1.131 =$ **0.80 m** (0.7958). Change 900 litres into 0.9 m³, then divide by the area of the base, $\pi \times 0.6^2$. The water is 0.80 m deep.
d. $\pi \times 0.36 \times 0.01 = 0.01131$ m³ = **11 litres**. Change 1 cm into 0.01 m, then multiply by the base area. One centimetre of rain adds about 11 litres.

Answer to Question 12

a. The two semicircles make one full circle: $2\pi \times 36.5 = 73\pi = 229.3$. Lap $= 200 + 229.3 =$ **429.3 m**.
b. $5000 \div 429.3 = 11.65$, so she needs **12 laps** to run at least 5000 m, which is $12 \times 429.3 = 5151.6$ m.
c. Rectangle $100 \times 73 = 7300$. Circle $\pi \times 36.5^2 = 4185.4$. Total $= 11485.4$, so **11 485 m²**. The inside of the track is a rectangle 100 m by 73 m plus two semicircles that make one circle.
d. $11\,485 \times 3.20 =$ **£36 752**. So it costs about £36 752 to turf the inside of the track.

Answer to Question 13

a. $2 \times (5 + 4) \times 2.5 =$ **45 m²**. Two walls are 5 m by 2.5 m and two are 4 m by 2.5 m, so the total is the perimeter of the floor times the height.
b. Window $= 1.5 \times 1.2 = 1.8$. Painted area $= 45 - 2 - 1.8 =$ **41.2 m²**. The door and the window are not painted, so their areas are subtracted.
c. Two coats: 82.4 m². Litres $= 82.4 \div 12 =$ **6.9 litres** (6.87). Two coats means the area is doubled: $2 \times 41.2 = 82.4$ m². About 6.9 litres of paint are needed.
d. $6.87 \div 2.5 = 2.7$, so **3 tins**. Cost $= 3 \times 22 =$ **£66**. We must round up because 2 tins would not be enough. So 3 tins are needed and the cost is £66.

Answer to Question 14

a. A: $\pi \times 16 \times 10 = 160\pi = 503$ cm³. B: $\pi \times 25 \times 7 = 175\pi = 550$ cm³.
b. A: $89 \div 5.03 = 17.7$p per 100 cm³ (**18p**). B: $95 \div 5.50 = 17.3$p per 100 cm³ (**17p**).
c. **Tin B** is slightly better value (about 17.3p against 17.7p per 100 cm³). The smaller cost per 100 cm³ is the better buy, so tin B is better value.
d. $\pi \times 25 \times h = 160\pi$, so $h = 160 \div 25 =$ **6.4 cm**.

Answer to Question 15

a. $\frac{140}{360} \times \pi \times 64 = 24.89\pi =$ **78.2 m²**. The sector is $\frac{140}{360}$ of a circle of radius 8 m. The watered area is 78.2 m².

b. $\frac{140}{360} \times 2\pi \times 8 = \frac{140}{360} \times 16\pi = 19.5$, so **19.5 m**. The fence follows the arc, which is $\frac{140}{360}$ of the circumference 16π. The fence is 19.5 m long.

c. $19.5 + 8 + 8 =$ **35.5 m**. The two straight edges are radii of 8 m each. The perimeter of the watered region is 35.5 m.

d. $\frac{\theta}{360} \times 64\pi = 100$, so $\theta = \frac{100 \times 360}{64\pi} = 179.0$, which is **179°**. Rearrange the sector area formula to make θ the subject. The sprinkler must turn through about 179°.

Answer to Question 16

a. $l = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$ cm. $V = \frac{1}{3}\pi \times 25 \times 12 = 100\pi =$ **314.2 cm³**. The radius, the height and the slant height form a right-angled triangle, so we use Pythagoras. The slant height is 13 cm and the volume is 314.2 cm³.

b. $\pi \times 5 \times 13 = 65\pi$ **cm² = 204.2 cm²**. Use the slant height $l = 13$, not the vertical height. The curved surface area is 65π cm², about 204.2 cm².

c. $V = \frac{4}{3}\pi \times 216 = 288\pi =$ **904.8 cm³**. $A = 4\pi \times 36 = 144\pi =$ **452.4 cm²**. Cube the radius for the volume and square it for the area. The sphere has volume 904.8 cm³ and surface area 452.4 cm².

d. $\frac{1}{3} \times 36 \times 9 =$ **108 cm³**. The base area is $6 \times 6 = 36$ cm². The volume of the pyramid is 108 cm³.

Answer to Question 17

a. $\pi \times 2^2 \times 5 = 20\pi$ m³. Use $V = \pi r^2 h$ with $r = 2$ and $h = 5$. The volume of the cylinder is 20π m³.

b. $\frac{1}{2} \times \frac{4}{3}\pi \times 8 = \frac{16}{3}\pi$ m³. A sphere of radius 2 m has volume $\frac{4}{3}\pi \times 8 = \frac{32}{3}\pi$, and the hemisphere is half of this. Its volume is $\frac{16}{3}\pi$ m³.

c. $20\pi + \frac{16}{3}\pi = \frac{76}{3}\pi = 79.6$ m³ = **79 587 litres** (about 79 600). Add the two exact volumes, then multiply by 1000 to change m³ into litres. The total volume is 79.6 m³.

d. Cylinder: $2\pi \times 2 \times 5 = 20\pi$. Hemisphere: $\frac{1}{2} \times 4\pi \times 4 = 8\pi$. Total $= 28\pi = 87.96$, so **88 m²**. The flat base and the circle where the two parts join are not painted. The area painted is about 88 m².

Answer to Question 18

a. $6^3 = 216$ **cm³**.
b. $\pi \times 4 \times h = 216$, so $h = 216 \div (4\pi) =$ **17.2 cm**. The volume does not change when the metal is melted, so the cylinder also has volume 216 cm³.
c. One sphere: $\frac{4}{3}\pi = 4.189$. $216 \div 4.189 = 51.57$, so **51 spheres**. We round down because only complete spheres count. So 51 spheres can be made.
d. Some metal is always wasted or lost when it is melted, and the spheres would never use up the volume perfectly, so fewer would be made.

Answer to Question 19

a. $\pi \times 10^2 = 100\pi$ **cm²**.
b. A square with diagonal d has area $\frac{1}{2}d^2$. With $d = 20$: $\frac{1}{2} \times 400 =$ **200 cm²**.
c. $100\pi - 200 = 114.2$, so $100\pi - 200$ **cm²** $\approx$ **114.2 cm²**. The shaded area is the area of the circle minus the area of the inner square.
d. The outer square has side 20 cm and area 400 cm². The circle covers $\frac{100\pi}{400} = \frac{\pi}{4} = 78.5\%$ (1 d.p.).

Answer to Question 20

a. 30 cm: $\pi \times 15^2 = 225\pi = 706.9$ cm². 20 cm: $\pi \times 10^2 = 100\pi = 314.2$ cm².
b. 30 cm: $12 \div 706.9 =$ **0.0170** £ per cm². 20 cm: $7 \div 314.2 =$ **0.0223** £ per cm².
c. The **30 cm pizza** is better value. Two 20 cm pizzas give 628.3 cm² for £14, which is less pizza for more money than the 30 cm pizza (706.9 cm² for £12).
d. Area needed $= 2 \times 100\pi = 200\pi$. Then $\pi r^2 = 200\pi$, so $r = \sqrt{200} = 14.14$ and the diameter is **28.3 cm** (so a 28.3 cm pizza equals two 20 cm pizzas).