Lesson - Angles, polygons and properties of shapes

Year: Year 10

Angle facts and the properties of shapes are tested on every GCSE paper, often with a requirement to give **reasons**. You need the angle rules, the angles in polygons, the properties of quadrilaterals and circles, and the conditions for congruent triangles. These topics are on both Foundation and Higher tiers.

Starter activity

Look around you and find five different polygons in the classroom or at home: a window, a book, a floor tile, a road sign, a ceiling light. Name each one and say whether it looks regular. Tape or draw a triangle on paper, cut off its three corners and place the corners side by side along a straight edge. What do you notice about the angle where they meet? Does it work for a triangle of any shape? Try a quadrilateral and its four corners round a point.

Basic angle facts

Key fact: basic angle facts

- Angles on a straight line add up to 180°.
- Angles around a point add up to 360°.
- Vertically opposite angles are equal.
- Angles in a triangle add up to 180°; in a quadrilateral 360°.
- An isosceles triangle has two equal sides and two equal base angles. An equilateral triangle has three angles of 60°.

Worked example 1

angles on a straight line 112° and x: $x = 180 - 112 = 68^\circ$.

Isosceles triangle with apex 40°: base angles $\frac{180-40}{2} = 70^\circ$ each.

Angles and parallel lines

Rule: angles and parallel lines

- **Alternate angles** (Z shape) are equal.
- **Corresponding angles** (F shape) are equal.
- **Co-interior angles** (C or U shape) add up to 180°.

If one angle is 65°, its alternate and corresponding angles are 65° and its co-interior

angle is $180 - 65 = 115^\circ$. In an exam write the reason in words, for example "alternate angles are equal", after each angle you find.

Angles in polygons

Definition: interior and exterior angles

An **interior angle** is inside a polygon at a vertex. An **exterior angle** is formed by extending one side. Interior + exterior $= 180^\circ$.

Rule: angles in polygons

A polygon with n sides can be split into $n - 2$ triangles from one vertex, so the **sum of interior angles** is $(n - 2) \times 180^\circ$. A hexagon: $4 \times 180 = 720^\circ$. A pentagon: 540°. A heptagon: 900°.

The **exterior angles** of any polygon add up to 360°. For a **regular** polygon (equal sides and angles): exterior angle $= \dfrac{360}{n}$ and interior angle $= 180 - \text{exterior}$. A regular octagon has exterior angle 45° and interior angle 135°. A regular polygon with exterior angle 20° has $\frac{360}{20} = 18$ sides. If the interior angle is 156°, the exterior angle is 24° and $n = 15$.

Properties of quadrilaterals

Key fact: properties of quadrilaterals

- **Parallelogram:** opposite sides parallel and equal; opposite angles equal; angles next to each other add to 180°.
- **Rectangle:** four right angles; diagonals equal and bisect each other.
- **Rhombus:** four equal sides; diagonals bisect at right angles.
- **Square:** a rectangle and rhombus at once.
- **Kite:** two pairs of adjacent equal sides; one pair of equal angles; diagonals cross at right angles.
- **Trapezium:** one pair of parallel sides.

Congruent triangles

Definition: congruent shapes

Shapes are **congruent** if they are identical in shape and size. They may be rotated or reflected.

Rule: conditions for congruent triangles

Congruent shapes are identical in shape and size (they may be rotated or reflected). Triangles are congruent if one of these holds:

- **SSS:** three sides equal.
- **SAS:** two sides and the *included* angle equal.
- **ASA** (or AAS): two angles and a corresponding side equal.
- **RHS:** right angle, hypotenuse and one other side equal.

"AAA" is *not* a condition: three equal angles give similar triangles, not necessarily congruent ones.

Circle vocabulary

Definition: circle vocabulary

The **radius** goes from the centre to the circumference. The **diameter** passes through the centre (twice the radius) and is the longest chord. A **chord** joins two points on the circumference. A **tangent** touches the circle at one point. An **arc** is part of the circumference. A **sector** is a slice bounded by two radii and an arc, and a **segment** is bounded by a chord and an arc.

Forming equations with angles

Method: forming equations with angles

Angles in a triangle are x, $2x$ and $3x + 30$ degrees. $6x + 30 = 180$, so $x = 25$ and the angles are 25°, 50° and 105°. An exterior angle of a triangle equals the sum of the two opposite interior angles: if the interior angles are a, b, c then $a + b + c = 180$ and the exterior angle at C is $180 - c = a + b$.

More worked examples in exam style

Worked example 2

In triangle ABC, $AB = AC$ and angle $BAC = 48^\circ$. Work out angle ABC [3 marks].

The triangle is isosceles, so the base angles are equal. $\frac{180-48}{2} = 66^\circ$. Reason: base angles of an isosceles triangle are equal and angles in a triangle add up to 180°.

Worked example 3

Work out the size of each exterior angle and each interior angle of a regular 9-sided polygon [3 marks].

Exterior $= \frac{360}{9} = 40^\circ$. Interior $= 180 - 40 = 140^\circ$.

Worked example 4

A parallelogram $PQRS$ has angle $P = 3x$ and angle $Q = 2x + 20$. Find x [3 marks].

Adjacent angles add to 180°: $5x + 20 = 180$, $x = 32$. Angle $P = 96^\circ$ and angle $Q = 84^\circ$.

Common mistakes

Watch out

- Using $n \times 180$ instead of $(n - 2) \times 180$ for interior angles.
- Mixing up alternate and co-interior angles.
- Giving the exterior angle when the interior is asked for.
- Writing "Z angles" or "F angles" as reasons: use the proper names.
- Using AAA as a congruence condition.

Exam technique

Exam technique

- Never say "it looks like"; every angle must have a reason.
- Use the full reason wording: "angles on a straight line add up to 180 degrees".
- In "show that" and proof questions, write the conclusion in words.

Reasoning with angles

GCSE questions rarely ask for a bare number. Most of them say “give a reason” or “show that”, and the marks are given for the method and the reason as well as the answer. A good reason uses the correct maths vocabulary: angles on a straight line add up to 180°, angles round a point add up to 360°, vertically opposite angles are equal, alternate angles are equal, corresponding angles are equal, co-interior angles add up to 180°, angles in a triangle add up to 180°, and base angles of an isosceles triangle are equal. Avoid vague words such as “Z angles” or “F angles”. Examiners accept them only when the proper name is also given.

Worked example 5: bearings and back bearings

The bearing of B from A is 065°. Find the bearing of A from B [2 marks].

Draw a north line at each point. The north lines are parallel, so the angle between AB and the north line at B is a co-interior angle with 65°, which gives $180 - 65 = 115^\circ$ on one side. The bearing is measured clockwise from north all the way round, so the bearing of A from B is $180 + 65 = 245^\circ$. A quick rule: if the bearing is less than 180° add 180°, and if it is greater than 180° subtract 180°.

Worked example 6: finding the number of sides

Each interior angle of a regular polygon is 140°. How many sides has it? [3 marks]

The exterior angle is $180 - 140 = 40^\circ$ because interior and exterior angles lie on a straight line. The exterior angles of any polygon add up to 360°, so the number of sides is $360 \div 40 = 9$. Check by finding the interior angle of a regular 9-sided polygon: $(9 - 2) \times 180 \div 9 = 1260 \div 9 = 140^\circ$. This agrees.

Worked example 7: an algebraic angle problem

In a parallelogram, two adjacent angles are $(3x + 15)^\circ$ and $(2x + 25)^\circ$. Find x and all four angles [4 marks].

Adjacent angles in a parallelogram are co-interior, so they add up to 180°: $3x + 15 + 2x + 25 = 180$. Collect like terms: $5x + 40 = 180$, so $5x = 140$ and $x = 28$. The angles are $3 \times 28 + 15 = 99^\circ$ and $2 \times 28 + 25 = 81^\circ$. Opposite angles are equal, so the four angles are 99°, 81°, 99°, 81°, which add up to 360°.

Worked example 8: congruence reasoning

ABCD is a rectangle and the diagonals AC and BD cross at E. Prove that triangles ABE and DCE are congruent [4 marks].

In a rectangle, opposite sides are equal, so $AB = DC$. The diagonals of a rectangle are equal and bisect each other, so $AE = DE$ and $BE = CE$. All three pairs of corresponding sides are equal, so the triangles are congruent by SSS. Always write the reason next to each statement, and name the triangles in matching order so that the corresponding vertices line up.

Note: facts to know about quadrilaterals

A square has four equal sides and four right angles, and its diagonals bisect each other at right angles. A rectangle has four right angles and equal diagonals. A rhombus has four equal sides and diagonals that bisect the angles. A parallelogram has opposite sides parallel and equal and diagonals that bisect each other. A kite has two pairs of adjacent equal sides and one pair of equal angles, and its diagonals cross at right angles. A trapezium has exactly one pair of parallel sides. Quadrilaterals have angle sum 360°, so you can find a missing angle quickly.

Watch out: circle theorem reasons

For circle theorems the reason must name the theorem in words, for example “the angle at the centre is twice the angle at the circumference” or “opposite angles of a cyclic quadrilateral add up to 180°”. Always say which angle you are finding before you state the reason.

Did you know?

The ancient Greek mathematician Euclid wrote a famous book called *The Elements* around 300 BC, in which every fact about angles and shapes is proved from a few simple starting rules. One of those rules, the parallel postulate, says that through a point not on a line there is exactly one line parallel to it. Mathematicians tried for 2000 years to prove it from the other rules, and in the 1800s they discovered that you can build different, perfectly good geometries without it. The 360 degrees in a circle probably come from the Babylonians, who counted in 60s and may have noticed that the Sun takes about 360 days to travel round the sky.

Otto's tip

Add every angle you find to the diagram as you go. Write the reason beside it in 3 or 4 words. Mark parallel lines with arrows and equal sides with dashes.