Worksheet - Angles, polygons and properties of shapes

Year: Year 10 · 20

Show your working.

Question 1

Question 1 – Basic angle facts *(Section A · Skills practice)*

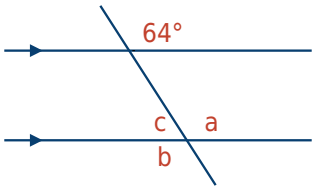
Non-calculator. Work out the size of each lettered angle. Give a reason for each answer.

a. Angles on a straight line are 128° and a.
b. Angles round a point are 95°, 110°, 80° and d.
c. Two angles of a triangle are 47° and 68°. Find the third angle e.
d. Two straight lines cross. One angle is 112°. Find the angle f next to it.

Question 2

Question 2 – Parallel lines *(Section A · Skills practice)*

In the diagram the two horizontal lines are parallel. The angle marked 64° is above the top line, to the right of the crossing line. Find a, b and c and give the special name of each pair of angles.



a. Angle a is in the matching position at the lower line.
b. Angle b is the alternate angle to 64°.
c. Angle c is between the lines on the left, and together with the vertically opposite angle of 64° forms a pair of co-interior angles.

Question 3

Question 3 – Regular polygons *(Section A · Skills practice)*

Non-calculator. Work out the size of one interior angle of each regular polygon by finding the exterior angle first.

a. a regular pentagon
b. a regular octagon
c. a regular 12-sided polygon

d. A regular polygon has exterior angles of 15°. How many sides has it, and what is each interior angle?

Question 4

Question 4 – Angle sums in polygons *(Section A · Skills practice)*

The sum of the interior angles of an n-sided polygon is $(n-2)\times 180^\circ$. Use it to answer these.

a. Find the angle sum of a hexagon.
b. Find the angle sum of a decagon (10 sides).
c. Find the angle sum of a 15-sided polygon.
d. The angles of a polygon add up to 2880°. How many sides has it?

Question 5

Question 5 – Types of quadrilateral *(Section A · Skills practice)*

Name the quadrilateral described each time, then answer the last part.

a. Four equal sides, with diagonals that cross at right angles but with angles that are not 90°.
b. Exactly one pair of parallel sides and the other two sides equal in length.
c. Two pairs of adjacent sides equal and one line of symmetry.
d. A parallelogram has one angle of 70°. Find the other three angles.

Question 6

Question 6 – Isosceles triangles and exterior angles *(Section A · Skills practice)*

Work out the lettered angles, and give a reason for each.

a. An isosceles triangle has an apex angle of 40°. Find each base angle.
b. An isosceles triangle has a base angle of 55°. Find the apex angle.
c. A triangle has interior angles of 50° and 65° at two corners. Find the exterior angle at the third corner.
d. An exterior angle of a triangle is 130° and one of the opposite interior angles is 55°. Find the other opposite interior angle.

Question 7

Question 7 – The roof truss *(Section B · Real-life practice)*

A roof truss is an isosceles triangle ABC with AB = AC and apex angle $A = 110^\circ$. A horizontal tie beam DE joins the two sloping rafters and is parallel to the base BC. A vertical strut runs from A to the midpoint M of BC.

a. Find the angle the rafter makes with the base, angle ABC.

b. What is the size of angle ADE (D on AB, E on AC)? Give the reason.
c. Find angle BAM.
d. Check that the angles in triangle ABM add up to 180°.

Question 8

Question 8 – Tessellating with polygons *(Section B · Real-life practice)*

Tiles of different regular polygons can fit together round a point only if their interior angles add up to exactly 360°.

Polygon	Interior angle
Triangle	60°
Square	90°
Pentagon	108°
Hexagon	120°
Octagon	135°

a. Show that three hexagons fit together at a point.
b. Show that two octagons and one square fit together at a point.
c. Show that three pentagons leave a gap and find the size of the gap.
d. Find how many equilateral triangles and how many squares can fit round a point if there are 3 triangles. (Hint: $3 \times 60 = 180$.)

Question 9

Question 9 – The stop sign *(Section B · Real-life practice)*

A road sign is a regular octagon with sides of 25 cm. Calculator allowed.

a. Find the size of each exterior angle and each interior angle.
b. Find the perimeter of the sign.
c. The sign is split into triangles by drawing lines from one corner to every other corner. How many triangles are there, and what is the sum of their angles?
d. Show that this sum equals the interior angle sum of an octagon.

Question 10

Question 10 – Sailing on bearings *(Section B · Real-life practice)*

A boat sails from a harbour A on a bearing of 050° to a buoy B. It then turns and sails on a bearing of 130° to a lighthouse C. The distances AB and BC are both 12 km. Bearings are measured clockwise from north.

a. Work out the back bearing of A from B.
b. Find angle ABC.
c. Use the fact that triangle ABC is isosceles to find angle BAC.

d. Find the bearing of C from A, and the bearing of A from C.

Question 11

Question 11 – Roads and paths *(Section B · Real-life practice)*

Two straight roads AB and CD are parallel. A cycle path crosses both of them. At the crossing with AB the path makes an angle of $2x + 10$ degrees with the road on one side, and at the crossing with CD it makes an angle of $x + 20$ degrees on the matching co-interior side.

a. Explain why $(2x + 10) + (x + 20) = 180$.
b. Solve the equation to find x.
c. Find the size of both angles.
d. The council plans a third road parallel to AB. How many angles of 70° will there be where the cycle path crosses all three roads (counting the 4 angles round each crossing)?

Question 12

Question 12 – The hexagonal window *(Section B · Real-life practice)*

A window frame is a regular hexagon with side 40 cm. Lines from the centre O to every corner split it into six identical triangles. Calculator allowed.

a. Find the angle at O in each triangle.
b. Show that each triangle is equilateral.
c. Find the perimeter of the window.
d. Each triangle has height $20\sqrt{3}$ cm. Work out the area of the window to the nearest cm².

Question 13

Question 13 – Forming equations with angles *(Section B · Real-life practice)*

Use algebra to find the angles in each shape. Show your equation each time.

a. The angles of a triangle are x, $2x + 10$ and $3x - 10$.
b. The angles of a quadrilateral are x, $x + 30$, $2x$ and $2x + 30$.
c. The angles of a pentagon are x, $2x$, $3x$, $2x - 10$ and 110.
d. Which of the shapes has an angle bigger than 150°?

Question 14

Question 14 – Designing a kite *(Section B · Real-life practice)*

Kite ABCD has AB = AD and CB = CD. Angle $A = 76^\circ$ and angle $C = 112^\circ$. The diagonals AC and BD cross at E.

a. Find angles B and D.
b. Find angle BAC and angle BCA, using the line of symmetry AC.
c. Find angle ABE and angle CBE, given that the diagonals cross at right angles.
d. Check that angle ABE + angle CBE = angle B.

Question 15

Question 15 – Proof: angle sum and exterior angle *(Section C · Problem solving)*

Triangle PQR has interior angles p at P, q at Q and r at R. A line is drawn through P parallel to QR. The line makes angles x and y with PQ and PR as shown (one each side of P, above the triangle).

a. Explain why $x = q$ and $y = r$.
b. Use angles on a straight line to prove that $p + q + r = 180^\circ$.
c. Side QR is extended beyond R. Prove that the exterior angle at R equals $p + q$.
d. Check with $p = 55^\circ$ and $q = 75^\circ$.

Question 16

Question 16 – Polygon puzzles *(Section C · Problem solving)*

Non-calculator. Each question is about a regular polygon.

a. Each interior angle is 8 times the exterior angle. Find the number of sides.
b. Each interior angle is 156°. Find the number of sides.
c. The interior angle is 100° bigger than the exterior angle. How many sides has the polygon?
d. Explain why no regular polygon has an interior angle of 100°.

Question 17

Question 17 – Congruent triangles *(Section C · Problem solving)*

Triangle ABC is isosceles with AB = AC. M is the midpoint of BC. The line AM is drawn.

a. Prove that triangles ABM and ACM are congruent. State the condition (SSS, SAS, ASA or RHS) you use.
b. Hence prove that angle ABC = angle ACB and that angle $AMB = 90^\circ$.
c. Which pair of triangles is congruent? Give the condition. (i) 5 cm, 7 cm, 9 cm and 9 cm, 5 cm, 7 cm; (ii) 6 cm, 8 cm with an angle of 40° between them, and 6 cm, 8 cm with an angle of 50° between them.
d. Explain why two triangles with all three angles equal need not be congruent.

Question 18

Question 18 – Circles and tangents *(Section C · Problem solving)*

O is the centre of a circle. TA and TB are tangents from a point T outside the circle, touching it at A and B. Angle $ATB = 40^\circ$.

a. Explain why angles OAT and OBT are both 90°.
b. Triangle TAB is isosceles (TA = TB). Find angle TAB.
c. Find angle OAB.
d. Find angle AOB and explain how you can check it using the quadrilateral OATB.

Question 19

Question 19 – Higher: circle theorems *(Section C · Problem solving)*

Higher tier. A, B, C and D lie on a circle with centre O. Use circle theorems and give a reason for each answer.

a. Angle $AOB = 108^\circ$ at the centre. Find angle ACB at the circumference on the same arc.
b. Angle $ABC = 78^\circ$ in cyclic quadrilateral ABCD. Find angle ADC.
c. AB is a diameter and C is on the circle. Angle $CAB = 34^\circ$. Find angle ABC.
d. A tangent at A makes an angle of 52° with the chord AB. Find the angle in the alternate segment.

Question 20

Question 20 – Challenge: the five-pointed star *(Section D · Challenge)*

A regular pentagon has each side extended until the lines meet, forming a five-pointed star (a pentagram) with a pentagon in the middle.

a. Show that the interior angle of the regular pentagon is 108°.
b. Each point of the star is an isosceles triangle on a side of the pentagon. Show that its base angles are 72°.
c. Find the angle at the tip of each point.
d. Show that the five tip angles add up to 180°.