Answers - Angles, polygons and properties of shapes

Year: Year 10

Answer to Question 1

a. Angles on a straight line add up to $180°$: $a = 180 - 128 =$ **52°**
b. Angles round a point add up to $360°$: $d = 360 - (95 + 110 + 80) = 360 - 285 =$ **75°**
c. Angles in a triangle add up to $180°$: $e = 180 - (47 + 68) = 180 - 115 =$ **65°**
d. Angles on a straight line: $f = 180 - 112 =$ **68°**. (The angle opposite $112°$ is also $112°$: vertically opposite angles are equal.) So $f = 68°$, because f and $112°$ lie on the same straight line.

Answer to Question 2

a. $a =$ **64°** (corresponding angles are equal). Corresponding angles sit in the same position at each parallel line (the F shape).
b. $b =$ **64°** (alternate angles are equal, the Z shape). So $a = b = 64°$, because both are matched with the given angle on parallel lines.
c. The angle vertically opposite $64°$ is $64°$, and co-interior angles add up to $180°$ (the C shape), so $c = 180 - 64 =$ **116°**. Check: $64 + 116 = 180$.

Answer to Question 3

a. Exterior $= 360 \div 5 = 72°$, interior $= 180 - 72 =$ **108°** The exterior and interior angles lie on a straight line, so they add up to $180°$. Each interior angle of a regular pentagon is $108°$.
b. Exterior $= 360 \div 8 = 45°$, interior $= 180 - 45 =$ **135°** Check with the angle sum: $(8 - 2) \times 180 \div 8 = 1080 \div 8 = 135°$.
c. Exterior $= 360 \div 12 = 30°$, interior $= 180 - 30 =$ **150°** So each interior angle of a regular 12-sided polygon is $150°$.
d. Number of sides $= 360 \div 15 =$ **24**. Interior $= 180 - 15 =$ **165°**. The exterior angles of any polygon add up to $360°$, so the number of sides is 360 divided by one exterior angle. The polygon has 24 sides and each interior angle is $165°$.

Answer to Question 4

a. $(6 - 2) \times 180 = 4 \times 180 =$ **720°** A hexagon has 6 sides, so it splits into 4 triangles from one corner. The angle sum of a hexagon is $720°$.

b. $(10-2)\times 180 = 8\times 180 =$ **1440°** A decagon splits into 8 triangles, each with angles adding up to 180°. The angle sum is 1440°.
c. $(15-2)\times 180 = 13\times 180 =$ **2340°** A 15-sided polygon splits into 13 triangles, so its interior angles add up to 2340°.
d. $(n-2)\times 180 = 2880$, so $n-2 = 16$ and $n =$ **18**. Divide both sides by 180: $2880 \div 180 = 16$. Then add 2. The polygon has 18 sides. Check: $16\times 180 = 2880$.

Answer to Question 5

a. A **rhombus**. All four sides are equal, so it is a rhombus; it is not a square because the angles are not right angles.
b. An **isosceles trapezium**. One pair of parallel sides makes it a trapezium, and the equal non-parallel sides make it isosceles.
c. A **kite**. Two pairs of equal adjacent sides and exactly one line of symmetry describe a kite.
d. Opposite angles are equal and co-interior angles add to 180°: the angles are **70°, 110°, 70°, 110°** (sum $= 360^\circ$). The angle next to 70° is $180-70 = 110^\circ$. The other two angles are opposite these, so they are 70° and 110°.

Answer to Question 6

a. $(180-40)\div 2 =$ **70°** each (base angles of an isosceles triangle are equal).
b. $180-2\times 55 =$ **70°** The two base angles are equal, so together they make 110°, and the apex angle is 70° because angles in a triangle add up to 180°.
c. The third interior angle is $180-50-65 = 65^\circ$, so the exterior angle is $180-65 =$ **115°**. This equals $50+65$: an exterior angle equals the sum of the two opposite interior angles.
d. $130-55 =$ **75°**. The exterior angle equals the sum of the two opposite interior angles, so the other interior angle is 75°.

Answer to Question 7

a. $(180-110)\div 2 =$ **35°** (isosceles triangle).
b. Angle ADE and angle ABC are corresponding angles on parallel lines, so $ADE =$ **35°**.
c. The strut bisects the apex angle in an isosceles triangle: $110\div 2 =$ **55°**.
d. $55+35+90 = 180$ (the strut is vertical, so angle $AMB = 90^\circ$). The angles do add up. So triangle ABM has angles of 55°, 35° and 90°.

Answer to Question 8

a. $3\times 120 =$ **360°**, so they fit. Three hexagons fill the space round the point exactly,

with no gap and no overlap.

b. $2 \times 135 + 90 = 270 + 90 =$ **360°**, so they fit. This is why floors are often tiled with octagons and small squares.
c. $3 \times 108 = 324^\circ$. The gap is $360 - 324 =$ **36°**, so pentagons alone do not tessellate.
d. $360 - 180 = 180$ and $180 \div 90 = 2$, so **2 squares**: 3 triangles and 2 squares ($3 \times 60 + 2 \times 90 = 360$). So 3 triangles and 2 squares fit round a point, because the angles add up to exactly 360°.

Answer to Question 9

a. Exterior $= 360 \div 8 =$ **45°**, interior $= 180 - 45 =$ **135°**. For any regular polygon the exterior angle is $360 \div n$, and the interior and exterior angles add up to 180°.
b. $8 \times 25 =$ **200 cm**. The perimeter is 8 sides of 25 cm, so the perimeter of the sign is 200 cm.
c. An octagon has $8 - 2 = 6$ triangles, with angle sum $6 \times 180 =$ **1080°**. From one corner we can draw lines to all corners except itself and its two neighbours, which makes 6 triangles. Their angles add up to 1080°.
d. $8 \times 135 = 1080^\circ$, which matches $(8 - 2) \times 180 = 1080^\circ$. So the angles of the 6 triangles make exactly the interior angles of the octagon, and the sum is 1080° both ways.

Answer to Question 10

a. Back bearing $= 50 + 180 =$ **230°**.
b. At B, the direction back to A is 230° and the direction to C is 130°, so angle $ABC = 230 - 130 =$ **100°**.
c. $(180 - 100) \div 2 =$ **40°**. The two base angles of the isosceles triangle are equal and add to $180 - 100 = 80^\circ$.
d. Bearing of C from A $= 50 + 40 =$ **090°** (due east). Bearing of A from C $= 90 + 180 =$ **270°** (due west).

Answer to Question 11

a. Co-interior angles between parallel lines add up to 180°.
b. $3x + 30 = 180$, so $3x = 150$ and $x =$ **50**. Check: $2 \times 50 + 10 = 110$ and $50 + 20 = 70$.
c. $2x + 10 =$ **110°** and $x + 20 =$ **70°**. (Sum $= 180$.)
d. At each crossing there are 2 angles of 70° (vertically opposite) and 2 of 110°, so over 3 crossings: **6 angles of 70°**.

Answer to Question 12

a. $360 \div 6 =$ **60°**.
b. The two other angles are equal (isosceles triangle) and add to 120°, so each is 60°. All three angles are 60°, so the triangle is **equilateral**.
c. $6 \times 40 =$ **240 cm**. The window has 6 sides of 40 cm, so the perimeter is 240 cm.
d. One triangle: $\frac{1}{2} \times 40 \times 20\sqrt{3} = 400\sqrt{3} = 692.8$ cm². Six triangles: $6 \times 692.82 =$ **4157 cm²** (nearest cm²).

Answer to Question 13

a. $x + 2x + 10 + 3x - 10 = 180$, $6x = 180$, $x = 30$. Angles: **30°, 70°, 80°**. The angles in a triangle add up to 180°. Check: $30 + 70 + 80 = 180$.
b. $6x + 60 = 360$, $x = 50$. Angles: **50°, 80°, 100°, 130°**. The angles in a quadrilateral add up to 360°: $x + x + 30 + 2x + 2x + 30 = 6x + 60$. Check: $50 + 80 + 100 + 130 = 360$.
c. $8x + 100 = 540$, $x = 55$. Angles: **55°, 110°, 165°, 100°, 110°**. The angles of a pentagon add up to $(5 - 2) \times 180 = 540^\circ$. Check: $55 + 110 + 165 + 100 + 110 = 540$.
d. Only the **pentagon** (it has 165°). The triangle's largest angle is 80° and the quadrilateral's is 130°, so only the pentagon has an angle bigger than 150°.

Answer to Question 14

a. $B = D = (360 - 76 - 112) \div 2 = 172 \div 2 =$ **86°**. The angles of a quadrilateral add up to 360°, and in a kite the angles between the unequal sides, B and D, are equal.
b. AC is the line of symmetry, so $BAC = 76 \div 2 =$ **38°** and $BCA = 112 \div 2 =$ **56°**. The line of symmetry cuts angles A and C exactly in half.
c. In triangle ABE: $180 - 90 - 38 =$ **52°**. In triangle CBE: $180 - 90 - 56 =$ **34°**. Angle AEB is 90° because the diagonals of a kite cross at right angles.
d. $52 + 34 = 86$, which equals angle B. The check works, so the answers are consistent.

Answer to Question 15

a. x and q are alternate angles between parallel lines, so $x = q$. In the same way y and r are alternate angles, so $y = r$.
b. Angles on a straight line at P: $x + p + y = 180^\circ$. Replace x by q and y by r: $q + p + r = 180^\circ$. So **the angles of a triangle add up to** 180°.
c. Exterior angle at R $= 180 - r$ (straight line). From part (b), $p + q = 180 - r$. So **the exterior angle** $= p + q$.
d. Third angle $r = 180 - 55 - 75 = 50^\circ$. Exterior angle $= 180 - 50 = 130^\circ = 55 + 75$.

Answer to Question 16

a. Let the exterior angle be e. Then $180 - e = 8e$, $9e = 180$, $e = 20$, and $n = 360 \div 20 =$ **18**.
b. Exterior $= 180 - 156 = 24^\circ$, so $n = 360 \div 24 =$ **15**.
c. $180 - e = e + 100$, so $2e = 80$, $e = 40$, $n = 360 \div 40 =$ **9**.
d. The exterior angle would be 80° and $360 \div 80 = 4.5$, which is not a whole number, so **no regular polygon has an interior angle of** 100°.

Answer to Question 17

a. AB = AC (given), BM = CM (M is the midpoint), AM is common to both. All three pairs of sides are equal, so the triangles are congruent by **SSS**.
b. Corresponding angles in congruent triangles are equal, so angle $ABM =$ angle ACM. The angles AMB and AMC are equal and lie on a straight line, so each is $180 \div 2 =$ **90°**.
c. Pair (i) is congruent by **SSS** (the same three lengths). Pair (ii) is not congruent because the included angles differ.
d. The triangles could be different sizes, for example an equilateral triangle of side 3 cm and one of side 6 cm. Equal angles give similar triangles, not congruent ones.

Answer to Question 18

a. A tangent meets the radius at 90°. So in triangles OAT and OBT the angles at A and B are right angles, because each tangent is perpendicular to the radius drawn to its point of contact.
b. $(180 - 40) \div 2 =$ **70°**. The two equal tangents make the base angles of triangle TAB equal, so each is 70°.
c. $OAB = OAT - TAB = 90 - 70 =$ **20°**. Angle OAT is split into angle OAB and angle TAB.
d. Triangle OAB is isosceles (OA = OB), so $AOB = 180 - 2 \times 20 =$ **140°**. Check in quadrilateral OATB: $90 + 90 + 40 + 140 = 360^\circ$. The angles of a quadrilateral add up to 360°, so the answer $AOB = 140^\circ$ is confirmed.

Answer to Question 19

a. The angle at the centre is twice the angle at the circumference: $108 \div 2 =$ **54°**.
b. Opposite angles of a cyclic quadrilateral add to 180°: $180 - 78 =$ **102°**.
c. The angle in a semicircle is 90°, so $ACB = 90^\circ$ and $ABC = 180 - 90 - 34 =$ **56°**.
d. By the alternate segment theorem, the angle in the alternate segment is **52°**.

Answer to Question 20

a. $(5-2) \times 180 \div 5 = 540 \div 5 =$ **108°**. The angles of a pentagon add up to 540° and in a regular pentagon all five are equal.
b. The base angle lies on a straight line with an interior angle: $180 - 108 =$ **72°**. Each extended side makes a straight line, so the angle outside the pentagon is $180 - 108 = 72^\circ$. The triangle is isosceles, so both base angles are 72°.
c. Tip angle $= 180 - 2 \times 72 =$ **36°**. The angles of each point add up to 180°, so the tip angle is 36°.
d. $5 \times 36 =$ **180°**. So the five tip angles of the star add up to 180°, the same as the angles of a single triangle.