Test answers - Algebraic notation, expanding and factorising

Year: Year 10 · Duration: 45 minutes · /20

Answer to Question 1 (4 marks)

a. $3 \times 4 - 2 \times (-3) = 12 + 6 =$ **18** (1 for substitution, 1 for the answer) Substitute with brackets: $-2 \times (-3) = +6$. So $3p - 2q = 18$.
b. $3x^2 + 7x$ (1 for each correct term) Collect the x^2 terms ($5x^2 - 2x^2 = 3x^2$) and the x terms ($3x + 4x = 7x$). The answer is $3x^2 + 7x$ (2 marks).

Check (a) another way: $3p = 12$ and $2q = -6$, and subtracting a negative is the same as adding, so $12 - (-6) = 18$. A common error is to write $12 - 6 = 6$, which loses both marks. In (b), x^2 and x are unlike terms, so $3x^2 + 7x$ cannot be simplified any further.

Answer to Question 2 (4 marks)

a. $x^2 - 3x + 7x - 21 = x^2 + 4x - 21$ (1 for four terms, 1 for the answer) Collect the middle terms: $-3x + 7x = 4x$.
b. $(x - 3)(x - 5)$ (1) We need two numbers with product 15 and sum -8: -3 and -5. Check: $x^2 - 5x - 3x + 15$.
c. $3xy(2x - 3y)$ (1) The HCF of 6 and 9 is 3 and both terms contain x and y. Check: $3xy \times 2x = 6x^2y$.

In (a), test the expansion with $x = 1$: $(8)(-2) = -16$ and $1 + 4 - 21 = -16$, so the expansion is correct. In (c), taking out only 3 or only $3x$ is not fully factorised and scores 0.

Answer to Question 3 (4 marks)

a. $2(3x + 2) + 2(x + 4) = 6x + 4 + 2x + 8 = 8x + 12$ (1 for 2×sum, 1 for the simplified answer) The perimeter is two lengths plus two widths, so $2(3x + 2) + 2(x + 4) = 8x + 12$ metres, as required.
b. $8x + 12 = 52$, $x = 5$ (1). Cost $= 52 \times 5 =$ **£260** (1) Solve $8x = 40$, so $x = 5$. The fence is 52 m long, so the total cost of the fence is £260.

In (b), check the value of x: the length is $3 \times 5 + 2 = 17$ m and the width is $5 + 4 = 9$ m, and $2(17 + 9) = 52$ m, which matches the perimeter given.

Answer to Question 4 (4 marks)

a. $(2n + 3)^2 = 4n^2 + 12n + 9$ (1). $(2n - 3)^2 = 4n^2 - 12n + 9$ (1). Difference $= 24n$

(1). $24n = 8 \times 3n$, a multiple of 8 (1). Expanding both brackets and subtracting, the $4n^2$ and 9 terms cancel. Since $3n$ is a whole number, $(2n+3)^2 - (2n-3)^2$ is a multiple of 8 for every whole number n.

A numerical check is useful but is not a proof: with $n = 1$, $25 - 1 = 24 = 8 \times 3$, and with $n = 2$, $49 - 1 = 48 = 8 \times 6$. Only the algebra, which works for every n, earns the marks. Alternatively, use the difference of two squares:
$(2n + 3 + 2n - 3)(2n + 3 - 2n + 3) = 4n \times 6 = 24n$.

Answer to Question 5 (4 marks)

a. $ac = -24$, numbers 3 and -8: $2x^2 + 3x - 8x - 12 = x(2x+3) - 4(2x+3) = (2x+3)(x-4)$ (1 for split, 1 for the answer) We need two numbers with product $2 \times (-12) = -24$ and sum -5. Check: $2x^2 - 8x + 3x - 12 = 2x^2 - 5x - 12$.

b. $x = -1.5$ **or** $x = 4$ (1) Set each bracket equal to zero: $2x + 3 = 0$ gives $x = -1.5$ and $x - 4 = 0$ gives $x = 4$.

c. $\dfrac{(x-4)(x+4)}{(x+4)(x-3)} = \dfrac{x-4}{x-3}$ (1) The top is a difference of two squares and the bottom factorises with 4 and -3; the common factor $(x+4)$ cancels.