Worksheet - Algebraic notation, expanding and factorising

Year: Year 10 · 20

Show your working.

Question 1

Question 1 – Substituting into expressions *(Section A · Skills practice)*

Non-calculator. Given that $a = 3$, $b = -2$ and $c = 5$, work out the value of each expression. Write negative numbers in brackets when you substitute.

a. $4a - b$
b. $ab + c$
c. $a^2 - b^2$
d. $2c^2 - 3ab$

Question 2

Question 2 – Simplifying expressions *(Section A · Skills practice)*

Simplify each expression by collecting like terms. Then say which one cannot be simplified any further.

a. $5x + 3y - 2x + 4y$
b. $3a^2 + 2a - a^2 + 7a$
c. $4pq + 3p - pq + p$
d. $2x + 3y + 5$

Question 3

Question 3 – Expanding single brackets *(Section A · Skills practice)*

Expand and simplify where possible.

a. $5(2x - 3)$
b. $-3(x + 4)$
c. $x(x^2 - 7)$
d. $4(x + 2) - 3(x - 1)$

Question 4

Question 4 – Expanding double brackets *(Section A · Skills practice)*

Expand and simplify. Show all four multiplications for part (a).

a. $(x+4)(x+5)$
b. $(x-3)(x+8)$
c. $(2x+1)(3x-4)$
d. $(x-6)^2$

Question 5

Question 5 – Factorising with a common factor *(Section A · Skills practice)*

Factorise each expression fully. Check each answer by expanding.

a. $12x + 18$
b. $x^2 - 9x$
c. $6a^2b + 15ab^2$
d. $20x^3 - 8x^2$

Question 6

Question 6 – Factorising quadratics *(Section A · Skills practice)*

Factorise each quadratic expression. For part (d) use the difference of two squares.

a. $x^2 + 7x + 12$
b. $x^2 - x - 20$
c. $x^2 - 10x + 21$
d. $x^2 - 49$

Question 7

Question 7 – Taxi fares *(Section B · Real-life practice)*

A taxi firm in Manchester uses the formula $C = 3.50 + 1.80m$ to work out the cost, C pounds, of a journey of m miles. Calculator allowed.

Journey	Distance
Home to station	4 miles
Station to airport	9.5 miles
Airport to hotel	14 miles

a. Work out the cost of each journey.
b. Explain what the 3.50 and the 1.80 represent in the formula.
c. A journey costs £21.50. How far was it?
d. Another firm uses $C = 2 + 2m$. For what distance do the two firms charge the same?

Question 8

Question 8 – Fencing the garden *(Section B · Real-life practice)*

Priya is planning a rectangular vegetable bed. The length is $(2x + 3)$ metres and the width is $(x - 1)$ metres. Fencing costs £6 per metre.

a. Write an expression for the perimeter in its simplest form.
b. Write an expression for the area and expand it.
c. Find the perimeter and the area when $x = 5$.
d. Find the cost of the fencing when $x = 5$.

Question 9

Question 9 – Phone contract *(Section B · Real-life practice)*

A mobile phone contract costs £12 a month plus 5p for every text message over the free allowance. Mo has the plan below.

Item	Cost
Monthly fee	£12
Extra text (each)	5p
Extra data pack (each)	£3

a. Write a formula for the monthly bill B pounds when Mo sends t extra texts and buys d data packs.
b. Work out B when $t = 40$ and $d = 2$.
c. In one month Mo's bill is £22. He bought 2 data packs. How many extra texts did he send?
d. Rearrange your formula to make t the subject.

Question 10

Question 10 – The number machine *(Section B · Real-life practice)*

Non-calculator. Ayesha and Ben play a number game. Ayesha says: think of a number, add 3, multiply by 4, then subtract 12.

a. Ben thinks of 7. What does he end up with?
b. Let Ben's number be n. Write the result as an expression in n and simplify it.
c. What do you notice? Explain why this happens.
d. Ayesha changes the game: think of a number, add 5, multiply by 3, subtract 9. Write the result as a simplified expression.

Question 11

Question 11 – Paving the patio *(Section B · Real-life practice)*

A patio is made from a large square of side $(x + 5)$ metres. A square flower bed of side x metres is left in one corner and is not paved.

a. Write an expression for the area of the large square and expand it.
b. Write an expression for the area to be paved and simplify.
c. Factorise your answer to (b) as a product of two brackets (use the difference of two squares).
d. Find the area to be paved when $x = 3$. Use your factorised form.

Question 12

Question 12 – School concert tickets *(Section B · Real-life practice)*

The school concert sells adult tickets at £a and child tickets at £c. On Friday the school sold 35 adult tickets and 52 child tickets. On Saturday it sold 48 adult tickets and 39 child tickets.

a. Write an expression for the money taken on Friday and another for Saturday.
b. Add them to write the total money taken in simplified form.
c. An adult ticket costs £6 and a child ticket costs £3.50. Find the total money taken over the two days.
d. Can your expression from (b) be factorised? Explain.

Question 13

Question 13 – Gift-wrap boxes *(Section B · Real-life practice)*

A gift box is a cuboid with a square base of side x cm and height $(x + 4)$ cm.

a. Write an expression for the volume of the box and expand it.
b. The surface area is $2x^2 + 4x(x + 4)$. Expand and simplify.
c. Factorise your surface area expression fully.
d. Find the surface area and volume when $x = 6$.

Question 14

Question 14 – Savings plan *(Section B · Real-life practice)*

Zara saves £x each week and her brother Leo saves £$(x + 2)$ each week. They both save for 10 weeks, and then Zara gets a £15 gift.

a. Write an expression for Zara's total savings including the gift and Leo's total savings.
b. Write an expression for the difference between them, simplified.
c. If Leo saves £6 a week, what is Zara's total including the gift?

Question 15

Question 15 – Prove it: consecutive numbers *(Section C · Problem solving)*

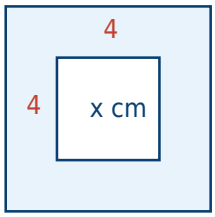
Mia says, “The sum of three consecutive whole numbers is always a multiple of 3.” Leo says, “The product of two consecutive whole numbers is always even.”

a. Let the three numbers be n, $n+1$, $n+2$. Prove Mia’s statement.
b. Let the two numbers be n and $n+1$. Explain why Leo is right using the fact that one of them is even.
c. Show that the difference between the squares of two consecutive numbers is always odd. Write the difference as $(n+1)^2 - n^2$.
d. Use part (c) to write 51 as the difference of two consecutive squares.

Question 16

Question 16 – The picture frame *(Section C · Problem solving)*

A square photo of side x cm is put in a square frame with a border of 4 cm on every side.



a. Write an expression for the side of the whole frame.
b. Write the area of the border as $(x+8)^2 - x^2$ and show that it simplifies to $16x + 64$.
c. Factorise $16x + 64$ fully.
d. The border has area 144 cm². Find x and the area of the photo.

Question 17

Question 17 – Difference of two squares *(Section C · Problem solving)*

Non-calculator. Use the factorisation $a^2 - b^2 = (a-b)(a+b)$ to answer these.

a. Factorise $x^2 - 81$, $4x^2 - 25$ and $9a^2 - 16b^2$.
b. Work out $63^2 - 37^2$ without a calculator by writing it as $(63-37)(63+37)$.
c. Work out $1.02^2 - 0.98^2$ the same way.
d. Factorise $x^4 - 1$ completely.

Question 18

Question 18 – Higher: factorising when a is not 1 *(Section C · Problem solving)*

Higher tier. Factorise each quadratic. Use the method: multiply $a \times c$, find two numbers with that product and the right sum, split the middle term, then factorise in pairs.

a. $2x^2 + 7x + 3$
b. $3x^2 - 10x + 8$
c. $6x^2 + x - 12$
d. Solve $2x^2 + 7x + 3 = 0$ using your answer to (a).

Question 19

Question 19 – Higher: algebraic fractions *(Section C · Problem solving)*

Higher tier. Simplify these algebraic fractions by factorising the top and the bottom first, then cancelling common factors.

a. $\dfrac{x^2 + 5x + 6}{x^2 - 4}$
b. $\dfrac{2x^2 + 6x}{x^2 - 9}$
c. $\dfrac{x^2 - x - 12}{x^2 + x - 6}$
d. Write $\dfrac{3}{x+1} + \dfrac{2}{x-2}$ as a single fraction.

Question 20

Question 20 – Challenge: magic squares of algebra *(Section D · Challenge)*

The sum of every row, column and diagonal in this magic square is the same.

$x + 3$	$x - 4$	$x + 1$
$x - 2$	x	$x + 2$
$x - 1$	$x + 4$	$x - 3$

a. Show that every row adds to $3x$.
b. Check one column and one diagonal.
c. Make a new magic square by multiplying every entry by 2 and adding 5. Write the middle entry and the magic sum in terms of x.
d. Explain why the new magic sum is $6x + 15$.