Answers - Algebraic notation, expanding and factorising

Year: Year 10

Answer to Question 1

a. $4 \times 3 - (-2) = 12 + 2 =$ **14** We replace a by 3 and b by (-2). Subtracting a negative number is the same as adding, so $4a - b = 14$.
b. $3 \times (-2) + 5 = -6 + 5 =$ **−1** Multiply first: a positive times a negative gives a negative, -6. Then add 5, so $ab + c = -1$.
c. $3^2 - (-2)^2 = 9 - 4 =$ **5** Both squares are positive because $(-2)^2 = 4$, not -4. So $a^2 - b^2 = 5$.
d. $2 \times 25 - 3 \times 3 \times (-2) = 50 + 18 =$ **68** Square before multiplying: $c^2 = 25$. The product $3ab = -18$, and subtracting -18 adds 18, so $2c^2 - 3ab = 68$.

Answer to Question 2

a. $5x - 2x = 3x$ and $3y + 4y = 7y$, so $3x + 7y$ Only terms with exactly the same letters can be collected.
b. $3a^2 - a^2 = 2a^2$ and $2a + 7a = 9a$, so $2a^2 + 9a$ a^2 and a are unlike terms, so they stay separate.
c. $4pq - pq = 3pq$ and $3p + p = 4p$, so $3pq + 4p$ pq and p are different terms, so the answer has two terms.
d. The terms $2x$, $3y$ and 5 are all unlike, so **part (d) cannot be simplified**. Only like terms can be added together.

Answer to Question 3

a. $5 \times 2x = 10x$ and $5 \times (-3) = -15$, so $10x - 15$ Each term inside the bracket is multiplied by 5.
b. $-3 \times x = -3x$ and $-3 \times 4 = -12$, so $-3x - 12$ The negative sign belongs to the 3, so both terms become negative.
c. $x^3 - 7x$ Multiply each term by x: $x \times x^2 = x^3$ and $x \times (-7) = -7x$, so the answer is $x^3 - 7x$.
d. $4x + 8 - 3x + 3 = x + 11$ Note that $-3 \times (-1) = +3$. Then collect: $4x - 3x = x$ and $8 + 3 = 11$, giving $x + 11$.

Answer to Question 4

a. $x^2 + 5x + 4x + 20 = x^2 + 9x + 20$ Every term in the first bracket multiplies every

term in the second, then the like terms $5x$ and $4x$ are collected.

b. $x^2 + 8x - 3x - 24 = x^2 + 5x - 24$ The middle terms give $8x - 3x = 5x$, and $-3 \times 8 = -24$.

c. $6x^2 - 8x + 3x - 4 = 6x^2 - 5x - 4$ Here $2x \times 3x = 6x^2$ and $1 \times (-4) = -4$; the middle terms give $-8x + 3x = -5x$.

d. $(x - 6)(x - 6) = x^2 - 6x - 6x + 36 = x^2 - 12x + 36$ A squared bracket means the bracket times itself, so there are two middle terms, not one: a common mistake is to write $x^2 + 36$.

Answer to Question 5

a. HCF is 6: $6(2x + 3)$ Check: $6 \times 2x + 6 \times 3 = 12x + 18$. So $12x + 18 = 6(2x + 3)$.

b. HCF is x: $x(x - 9)$ Both terms contain x, so x is taken outside. Check: $x \times x - 9x = x^2 - 9x$.

c. HCF is $3ab$: $3ab(2a + 5b)$ The HCF of 6 and 15 is 3, and both terms contain a and b. Check: $3ab \times 2a = 6a^2b$ and $3ab \times 5b = 15ab^2$.

d. HCF is $4x^2$: $4x^2(5x - 2)$ The HCF of 20 and 8 is 4, and the lowest power of x is x^2. The factorisation is complete because $5x - 2$ has no common factor.

Answer to Question 6

a. Two numbers with product 12 and sum 7: 3 and 4. $(x + 3)(x + 4)$ Check: $3 + 4 = 7$ and $3 \times 4 = 12$, so $x^2 + 7x + 12 = (x + 3)(x + 4)$.

b. Product -20, sum -1: -5 and 4. $(x - 5)(x + 4)$ Check: $-5 + 4 = -1$ and $-5 \times 4 = -20$, so $x^2 - x - 20 = (x - 5)(x + 4)$.

c. Product 21, sum -10: -3 and -7. $(x - 3)(x - 7)$ A positive product with a negative sum means both numbers are negative, so $x^2 - 10x + 21 = (x - 3)(x - 7)$.

d. $x^2 - 7^2 = (x - 7)(x + 7)$ There is no x term and 49 is a square number, so we use $a^2 - b^2 = (a - b)(a + b)$.

Answer to Question 7

a. 4 miles: $3.50 + 7.20 =$ **£10.70**. 9.5 miles: $3.50 + 17.10 =$ **£20.60**. 14 miles: $3.50 + 25.20 =$ **£28.70**. In each case multiply the miles by £1.80, then add the fixed charge of £3.50.

b. **£3.50** is the fixed starting charge and **£1.80** is the price for each mile. For example, a journey of 0 miles would still cost £3.50 and each extra mile adds £1.80.

c. $21.50 = 3.50 + 1.80m$, so $1.80m = 18$ and $m =$ **10 miles**. Subtract the fixed charge first, then divide by the price per mile: $18 \div 1.80 = 10$. Check: $3.50 + 18 = 21.50$.

d. $3.50 + 1.80m = 2 + 2m$ gives $1.50 = 0.2m$, so $m = 7.5$. The firms charge the same

for **7.5 miles** (£17). Check with the other firm: $2 + 2 \times 7.5 = 17$, and $3.50 + 1.80 \times 7.5 = 17$. Both firms charge £17.

Answer to Question 8

a. $P = 2(2x + 3) + 2(x - 1) = 4x + 6 + 2x - 2 = 6x + 4$ metres. Opposite sides are equal, so there are two lengths and two widths in the perimeter.
b. $A = (2x + 3)(x - 1) = 2x^2 - 2x + 3x - 3 = 2x^2 + x - 3$ m². Each term in the first bracket multiplies each term in the second, then $-2x + 3x = x$.
c. When $x = 5$: $P = 34$ m and $A = 50 + 5 - 3 =$ **52 m²**. (Check: length 13, width 4, $13 \times 4 = 52$.) Substituting into $6x + 4$: $6 \times 5 + 4 = 34$. So the perimeter is 34 m and the area is 52 m².
d. $34 \times 6 =$ **£204**. The fencing goes round the perimeter, so the total cost of the fencing is £204.

Answer to Question 9

a. $B = 12 + 0.05t + 3d$ Texts cost 5p, which is £0.05, so every amount in the formula is in pounds.
b. $B = 12 + 0.05 \times 40 + 3 \times 2 = 12 + 2 + 6 =$ **£20** Mo's bill for that month is therefore £20.
c. $22 = 12 + 0.05t + 6$, so $0.05t = 4$ and $t = 4 \div 0.05 =$ **80 texts**. The fixed parts are $12 + 3 \times 2 = 18$, so the texts cost $22 - 18 = 4$ pounds. Mo sent 80 extra texts.
d. $B - 12 - 3d = 0.05t$, so $t = \dfrac{B - 12 - 3d}{0.05}$ (or $20(B - 12 - 3d)$). Subtract 12 and $3d$ from both sides, then divide by 0.05. Dividing by 0.05 is the same as multiplying by 20.

Answer to Question 10

a. $7 + 3 = 10$, $10 \times 4 = 40$, $40 - 12 =$ **28**. Follow the steps in order, so Ben ends up with 28.
b. $4(n + 3) - 12 = 4n + 12 - 12 = 4n$. We put brackets around $n + 3$ because the whole result of the first step is multiplied by 4.
c. The result is always **4 times the starting number**. The $+12$ from the expansion cancels the -12 at the end. For example, starting with 7 gives $4 \times 7 = 28$, which matches part (a).
d. $3(n + 5) - 9 = 3n + 15 - 9 = 3n + 6$. Expand the bracket first, then collect the numbers: $15 - 9 = 6$. The result is $3n + 6$.

Answer to Question 11

a. $(x+5)^2 = x^2 + 5x + 5x + 25 = x^2 + 10x + 25$ m². The square has side $x+5$, so its area is $(x+5)(x+5)$; remember the two middle terms.
b. $x^2 + 10x + 25 - x^2 = 10x + 25$ m². The flower bed has area x^2, which is subtracted from the large square, so the area to be paved is $10x + 25$ m².
c. $(x+5)^2 - x^2 = (x+5-x)(x+5+x) = 5(2x+5)$, which is $5(2x+5)$ (the same as $10x+25$). Here $a = x+5$ and $b = x$. Expanding $5(2x+5)$ gives back $10x+25$, which checks the factorisation.
d. $5(6+5) =$ **55 m²**. When $x = 3$, $2x + 5 = 11$, so the area to be paved is $5 \times 11 = 55$ m². Check: $10 \times 3 + 25 = 55$.

Answer to Question 12

a. Friday: $35a + 52c$. Saturday: $48a + 39c$. Each adult ticket brings in £a and each child ticket brings in £c, so we multiply by the number of tickets sold.
b. $35a + 48a + 52c + 39c = 83a + 91c$ Collect the a terms and the c terms separately because they are unlike terms.
c. $83 \times 6 + 91 \times 3.5 = 498 + 318.50 =$ **£816.50** Substitute $a = 6$ and $c = 3.5$ into $83a + 91c$. The total money taken over the two days is £816.50.
d. No. The numbers 83 and 91 have no common factor greater than 1 (83 is prime and $91 = 7 \times 13$), so **the expression cannot be factorised**. To factorise, the two coefficients would need a common factor, and they do not.

Answer to Question 13

a. $V = x^2(x+4) = x^3 + 4x^2$ cm³. The volume of a cuboid is base area times height, and the base area is $x \times x = x^2$.
b. $2x^2 + 4x^2 + 16x = 6x^2 + 16x$ cm². Multiply out $4x(x+4) = 4x^2 + 16x$, then collect $2x^2 + 4x^2 = 6x^2$.
c. The HCF of $6x^2$ and $16x$ is $2x$: $2x(3x+8)$. Check by expanding: $2x \times 3x = 6x^2$ and $2x \times 8 = 16x$.
d. Surface area $= 2 \times 6 \times (18+8) = 312$ cm². Volume $= 216 + 144 = 360$ cm³. (Check: $6 \times 6 \times 10 = 360$.) Using $2x(3x+8)$ with $x = 6$: $12 \times 26 = 312$. So the surface area is 312 cm² and the volume is 360 cm³.

Answer to Question 14

a. Zara: $10x + 15$. Leo: $10(x+2) = 10x + 20$. Leo saves £$(x+2)$ for 10 weeks, so the whole bracket is multiplied by 10.
b. Leo − Zara $= 10x + 20 - (10x + 15) =$ **£5**. The difference does not depend on x. The brackets are important: the minus sign changes the sign of both $10x$ and 15. So Leo always has £5 more than Zara.
c. Leo saves £6, so $x + 2 = 6$ and $x = 4$. Zara: $10 \times 4 + 15 =$ **£55**. Check: Leo has $10 \times 6 = 60$ pounds, which is £5 more than Zara's £55.

Answer to Question 15

a. $n + (n+1) + (n+2) = 3n + 3 = 3(n+1)$, which is a multiple of 3 for every whole number n. Since $n+1$ is a whole number, $3(n+1)$ is 3 times a whole number. So the sum of three consecutive whole numbers is always a multiple of 3.
b. Of two consecutive numbers, exactly one is even, and an even number times any whole number is even, so $n(n+1)$ is **always even**. If n is even, the product is even; if n is odd, then $n+1$ is even, so the product is still even.
c. $(n+1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1$, which is **odd** (2 times a whole number plus 1). For example, $5^2 - 4^2 = 25 - 16 = 9 = 2 \times 4 + 1$. Since $2n$ is always even, $2n+1$ is always odd.
d. $2n + 1 = 51$ gives $n = 25$, so $26^2 - 25^2 = 676 - 625 = 51$. So $51 = 26^2 - 25^2$, which is the difference of two consecutive squares.

Answer to Question 16

a. The side is $x + 4 + 4 = x + 8$ cm. The border adds 4 cm on the left and 4 cm on the right of the photo.
b. $(x+8)^2 - x^2 = x^2 + 16x + 64 - x^2 = 16x + 64$. The area of the border is the area of the whole frame minus the area of the photo.
c. $16(x+4)$ The HCF of 16 and 64 is 16. Check: $16 \times x + 16 \times 4 = 16x + 64$.
d. $16x + 64 = 144$, so $16x = 80$ and $x = 5$. The photo is $5 \times 5 =$ **25 cm²**. Check: $(5+8)^2 - 5^2 = 169 - 25 = 144$. So $x = 5$ and the photo has area 25 cm².

Answer to Question 17

a. $(x-9)(x+9)$, $(2x-5)(2x+5)$, $(3a-4b)(3a+4b)$. In each case write both terms as squares: $81 = 9^2$, $4x^2 = (2x)^2$, $25 = 5^2$, $9a^2 = (3a)^2$ and $16b^2 = (4b)^2$.
b. $26 \times 100 =$ **2600**. Here $63 - 37 = 26$ and $63 + 37 = 100$, so $63^2 - 37^2 = 2600$. This is much quicker than squaring each number.
c. $(1.02 - 0.98)(1.02 + 0.98) = 0.04 \times 2 =$ **0.08**. Here $1.02 - 0.98 = 0.04$ and $1.02 + 0.98 = 2$, so $1.02^2 - 0.98^2 = 0.08$.
d. $x^4 - 1 = (x^2 - 1)(x^2 + 1) = (x-1)(x+1)(x^2+1)$. First treat x^4 as $(x^2)^2$. Then $x^2 - 1$ is also a difference of two squares, but $x^2 + 1$ cannot be factorised.

Answer to Question 18

a. $ac = 6$, numbers 1 and 6: $2x^2 + x + 6x + 3 = x(2x+1) + 3(2x+1) = (2x+1)(x+3)$ Check by expanding: $2x^2 + 6x + x + 3 = 2x^2 + 7x + 3$.
b. $ac = 24$, numbers -4 and -6: $3x^2 - 4x - 6x + 8 = x(3x-4) - 2(3x-4) = (3x-4)(x-2)$ The product is positive and the sum is negative, so both numbers

are negative. Check: $3x^2 - 6x - 4x + 8 = 3x^2 - 10x + 8$.

c. $ac = -72$, numbers 9 and -8: $6x^2 + 9x - 8x - 12 = 3x(2x + 3) - 4(2x + 3) = (2x + 3)(3x - 4)$ Check by expanding: $6x^2 - 8x + 9x - 12 = 6x^2 + x - 12$.

d. $(2x + 1)(x + 3) = 0$, so $x = -\frac{1}{2}$ **or** $x = -3$. If a product is zero, one of the brackets is zero: $2x + 1 = 0$ gives $x = -\frac{1}{2}$ and $x + 3 = 0$ gives $x = -3$.

Answer to Question 19

a. $\frac{(x+2)(x+3)}{(x-2)(x+2)} = \frac{x+3}{x-2}$ The common factor $(x + 2)$ cancels, so the fraction simplifies to $\frac{x+3}{x-2}$.

b. $\frac{2x(x+3)}{(x-3)(x+3)} = \frac{2x}{x-3}$ The top has common factor $2x$ and the bottom is a difference of two squares; $(x + 3)$ cancels.

c. $\frac{(x-4)(x+3)}{(x+3)(x-2)} = \frac{x-4}{x-2}$ Both quadratics contain the factor $(x + 3)$, which cancels top and bottom.

d. $\frac{3(x-2)+2(x+1)}{(x+1)(x-2)} = \frac{5x-4}{(x+1)(x-2)}$ The common denominator is $(x + 1)(x - 2)$. The numerator is $3x - 6 + 2x + 2 = 5x - 4$.

Answer to Question 20

a. Row 1: $x + 3 + x - 4 + x + 1 = 3x$. Row 2: $x - 2 + x + x + 2 = 3x$. Row 3: $x - 1 + x + 4 + x - 3 = 3x$. Every row adds up to $3x$ because the numbers in each row cancel out: $3 - 4 + 1 = 0$, $-2 + 0 + 2 = 0$ and $-1 + 4 - 3 = 0$.

b. Column 1: $x + 3 + x - 2 + x - 1 = 3x$. Diagonal: $x + 3 + x + x - 3 = 3x$. The column and the diagonal also add to $3x$, so the square is magic.

c. Middle: $2x + 5$. Magic sum: $3(2x + 5) = 6x + 15$. The middle entry x becomes $2 \times x + 5 = 2x + 5$, and the magic sum is three times the middle entry.

d. Each of the 3 numbers in a line is doubled and has 5 added, so the sum is $2 \times 3x + 3 \times 5 = 6x + 15$. So the new square is still magic, and its magic sum is $6x + 15$.