Test - Vectors and systems of linear equations

Year: University Year 1 · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Products of vectors

Let $\mathbf{a} = (1, 2, 2)$ and $\mathbf{b} = (3, 0, 4)$.

a. Find the angle between $\mathbf{a}$ and $\mathbf{b}$. (2 marks)
b. Find $\mathbf{a} \times \mathbf{b}$ and the area of the triangle with sides $\mathbf{a}$, $\mathbf{b}$. (2 marks)

Question 2 (4 marks)

Question 2 – A plane through three points

Find the equation of the plane through $A(1, 0, 0)$, $B(0, 2, 0)$, $C(0, 0, 3)$ and its distance from the origin.

Question 3 (4 marks)

Question 3 – Gaussian elimination

Solve $x + y + z = 2$, $2x + 3y + z = 1$, $x - y + 2z = 6$.

Question 4 (4 marks)

Question 4 – A system with a parameter

Consider $x + 2y = 3$ and $2x + ky = c$.

a. For which k is there exactly one solution? (1 mark)
b. For $k = 4$, find the values of c giving no solution and infinitely many solutions, and describe the lines. (3 marks)

Question 5 (4 marks)

Question 5 – Problem: a line and a plane

A laser beam follows $\mathbf{r} = (2, 1, 0) + t(1, 1, 1)$ and hits a mirror in the plane $x + 2y + z = 10$.

a. Find the point of impact. (2 marks)
b. Find the angle between the beam and the mirror. (2 marks)