Test - Vector spaces and linear maps

Year: University Year 1 · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Subspaces

Decide whether $U = \{(x, y, z) : x = 2y\}$ and $V = \{(x, y) : x \geq 0\}$ are subspaces. Justify.

Question 2 (4 marks)

Question 2 – Independence and basis

Show that $(1, 1, 1)$, $(1, 2, 3)$ and $(1, 4, 9)$ form a basis of $\mathbb{R}^3$, and find the coordinates of $(3, 7, 13)$.

Question 3 (4 marks)

Question 3 – A linear map

Let $T(x, y, z) = (x + y,\ y + z)$.

a. Write the matrix of T. (1 mark)
b. Find a basis of the kernel and of the image. (2 marks)
c. Verify rank-nullity. (1 mark)

Question 4 (4 marks)

Question 4 – Problem: the sensor

A 3×5 matrix A has rank 2 and represents a measurement system on $\mathbb{R}^5$.

a. Find the nullity. (1 mark)
b. Find the dimension of the image and say whether the system can produce every vector of $\mathbb{R}^3$. (2 marks)
c. Is the map injective? Justify. (1 mark)

Question 5 (4 marks)

Question 5 – Coordinates and polynomials

In P_2 with basis $\{1, x, x^2\}$ consider $p_1 = 1 + x$, $p_2 = x + x^2$ and $p_3 = 1 + x^2$.

a. Write the coordinate vectors and show that p_1, p_2, p_3 form a basis. (2 marks)
b. Express $q = 2 + 3x + x^2$ in terms of p_1, p_2, p_3. (2 marks)