Test - Sequences and series convergence

Year: University Year 1 · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – A limit from the definition

Let $a_n = \dfrac{2n-1}{n+1}$.

a. Find the limit L and show that $|a_n - L| = \dfrac{3}{n+1}$. (2 marks)

b. Prove from the definition that $a_n \to L$. (2 marks)

Question 2 (4 marks)

Question 2 – Sums of series

a. Find $\displaystyle\sum_{n=1}^{\infty} \frac{4}{(2n-1)(2n+1)}$. (2 marks)

b. Find $\displaystyle\sum_{n=0}^{\infty} 3\left(-\tfrac{1}{2}\right)^n$. (2 marks)

Question 3 (4 marks)

Question 3 – Convergence tests

a. Does $\displaystyle\sum \frac{n}{n^3+1}$ converge? (2 marks)

b. Does $\displaystyle\sum \frac{2^n}{n^2}$ converge? (2 marks)

Question 4 (4 marks)

Question 4 – An alternating series

Consider $\sum_{n\geq 1} \frac{(-1)^{n+1}}{n^2}$.

a. Show that it converges absolutely. (2 marks)
b. Estimate its sum with four terms and give a bound for the error. (2 marks)

Question 5 (4 marks)

Question 5 – Problem: a power series

Consider $f(x) = \sum_{n\geq 1} \frac{(x-1)^n}{n\,2^n}$.

a. Find the radius of convergence. (2 marks)
b. Find the interval of convergence, testing the endpoints. (2 marks)