Test - Probability and random variables

Year: University Year 1 · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Independent events

A and B are independent with $P(A) = 0.4$ and $P(B) = 0.35$.

a. Find $P(A \cup B)$. (2 marks)
b. Find $P(A \mid B^c)$ and $P(A^c \cap B^c)$. (2 marks)

Question 2 (4 marks)

Question 2 – Two machines

Machine 1 makes 70% of the bolts in a workshop, with 4% defective; machine 2 makes the rest, with 10% defective.

a. Find the probability that a bolt is defective. (2 marks)
b. A bolt is defective. Find the probability that it came from machine 2. (2 marks)

Question 3 (4 marks)

Question 3 – A discrete random variable

$P(X = 1) = 0.2$, $P(X = 2) = 0.5$, $P(X = 3) = 0.3$.

a. Find $E[X]$ and $\mathrm{Var}(X)$. (2 marks)
b. Find $E[5 - 2X]$ and $\mathrm{Var}(5 - 2X)$. (2 marks)

Question 4 (4 marks)

Question 4 – Binomial and Poisson

In a large batch, 10% of light bulbs are faulty. A sample of 20 bulbs is taken.

a. Find the probability of exactly 2 faulty bulbs using $B(20, 0.1)$. (2 marks)
b. Approximate the same probability using a Poisson distribution and comment. (2

marks)

Question 5 (4 marks)

Question 5 – Problem: machined parts

The length of a machined part is $L \sim N(30,\ 0.4^2)$ mm. Parts are accepted if $29.5 \leq L \leq 30.6$.

a. Find the probability that a part is accepted. (2 marks)
b. Parts are packed in boxes of 10. Find the probability that every part in a box is acceptable, and comment on the process. (2 marks)