Test - Number theory and discrete mathematics

Year: University Year 1 · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Euclid and Bezout

Use the Euclidean algorithm to find $\gcd(390, 72)$ and write it as $390x + 72y$.

Question 2 (4 marks)

Question 2 – Congruences

a. Solve $9x \equiv 6 \pmod{15}$. (2 marks)
b. Find $2^{50} \bmod 7$. (2 marks)

Question 3 (4 marks)

Question 3 – Counting

a. How many arrangements are there of the letters of STATISTICS? (2 marks)
b. A committee of 5 is chosen from 7 men and 6 women. How many committees contain at least 2 women? (2 marks)

Question 4 (4 marks)

Question 4 – Graphs

A graph has 7 vertices with degrees $1, 2, 2, 3, 3, 3, 4$.

a. How many edges does it have? (1 mark)
b. Can it be a tree? Explain. (1 mark)
c. Does it have an Euler trail? Explain. (2 marks)

Question 5 (4 marks)

Question 5 – Problem: a recurrence

A sequence of savings tokens satisfies $a_n = 4a_{n-1} - 4a_{n-2}$ with $a_0 = 1$, $a_1 = 4$.

a. Compute a_2 and a_3. (1 mark)
b. Solve the recurrence and check your formula against a_3. (3 marks)