Test - Multivariable calculus

Year: University Year 1 · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Partial derivatives and the tangent plane

Let $f(x,y) = x^2 e^y$.

a. Find f_x, f_y and f_{xy}. (2 marks)
b. Find the tangent plane to $z = f(x,y)$ at $(2, 0, 4)$. (2 marks)

Question 2 (4 marks)

Question 2 – Critical points

Find and classify the critical points of $f(x,y) = x^3 - 12x + y^2 - 4y$.

Question 3 (4 marks)

Question 3 – Lagrange multipliers

Find the maximum of $f = xy^2$ subject to $x + y = 3$, with $x, y \geq 0$.

Question 4 (4 marks)

Question 4 – Double integrals

a. Evaluate $\int_0^2 \int_0^1 (x + 2y)\, dx\, dy$. (2 marks)
b. Use polar coordinates to find $\iint_D \dfrac{dA}{1 + x^2 + y^2}$ over the unit disc. (2 marks)

Question 5 (4 marks)

Question 5 – Problem: errors in a box

A cuboid has measured sides $x = 4$ m, $y = 3$ m and $z = 2$ m, each with possible error

± 0.05 m. Its volume is $V = xyz$.

a. Find V and the partial derivatives at these values. (2 marks)
b. Estimate the maximum error in V using the total differential. (2 marks)