Test - Logic, sets and proof techniques

Year: University Year 1 · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Negation and contrapositive

a. Write the negation of "For every real x there is a real y with $y > x$". (2 marks)
b. State the contrapositive of "If n is divisible by 6, then n is even". (2 marks)

Question 2 (4 marks)

Question 2 – Sets and counting

In a group of 50 people, 30 speak French, 22 speak Spanish and 10 speak neither.

a. How many speak both languages? (3 marks)
b. How many speak exactly one language? (1 mark)

Question 3 (4 marks)

Question 3 – A direct proof

Prove that the sum of three consecutive integers is divisible by 3.

Question 4 (4 marks)

Question 4 – Proof by contradiction

Prove that there is no largest prime number.

Question 5 (4 marks)

Question 5 – Induction

Prove by induction that $\sum_{i=1}^{n}(2i-1) = n^2$ for all $n \geq 1$.