Multiple choice questions - Logic, sets and proof techniques

Year: University Year 1 · 10

Question 1 The negation of $\forall x\,(x > 2)$ is

- ◯ **A.** $\forall x\,(x \leq 2)$
- ◯ **B.** $\exists x\,(x \leq 2)$
- ◯ **C.** $\exists x\,(x$
- ◯ **D.** $\forall x\,(x$

Question 2 The contrapositive of $P \Rightarrow Q$ is

- ◯ **A.** $Q \Rightarrow P$
- ◯ **B.** $\neg P \Rightarrow \neg Q$
- ◯ **C.** $\neg Q \Rightarrow \neg P$
- ◯ **D.** $P \Rightarrow \neg Q$

Question 3 $P \Rightarrow Q$ is false exactly when

- ◯ **A.** P false, Q true
- ◯ **B.** P true, Q false
- ◯ **C.** both false
- ◯ **D.** both true

Question 4 $(A \cup B)^c$ equals

- ◯ **A.** $A^c \cup B^c$
- ◯ **B.** $A^c \cap B^c$
- ◯ **C.** $A \cap B$
- ◯ **D.** $A^c \setminus B$

Question 5 The number of subsets of a 5-element set is

- ◯ **A.** 10
- ◯ **B.** 25
- ◯ **C.** 32
- ◯ **D.** 120

Question 6 The function $f(x) = x^2$ on $\mathbb{R}$ is

- ◯ **A.** injective only
- ◯ **B.** surjective only
- ◯ **C.** bijective

- ◯ **D.** neither injective nor surjective

Question 7 To prove that $\sqrt{2}$ is irrational one usually uses

- ◯ **A.** induction
- ◯ **B.** contradiction
- ◯ **C.** a counterexample
- ◯ **D.** a truth table

Question 8 The inductive step assumes

- ◯ **A.** $P(k+1)$
- ◯ **B.** $P(1)$ only
- ◯ **C.** $P(k)$ for some $k \geq n_0$
- ◯ **D.** $P(n)$ for all n

Question 9 Which is a counterexample to “every prime is odd”?

- ◯ **A.** 3
- ◯ **B.** 9
- ◯ **C.** 2
- ◯ **D.** 1

Question 10 An equivalence relation must be

- ◯ **A.** reflexive, symmetric, transitive
- ◯ **B.** reflexive, antisymmetric, transitive
- ◯ **C.** symmetric and injective
- ◯ **D.** transitive and surjective

Answer grid - Logic, sets and proof techniques

Year: University Year 1

Question	Answer	Explanation
1	**B**	Swap $\forall$ to $\exists$ and negate $x > 2$ to $x \leq 2$.
2	**C**	The contrapositive negates and reverses.
3	**B**	A true hypothesis with a false conclusion is the only failure.
4	**B**	De Morgan's law for sets.
5	**C**	$2^5 = 32$.
6	**D**	$f(2) = f(-2)$ and -1 is not attained.
7	**B**	Assume $\sqrt{2} = p/q$ in lowest terms and derive a contradiction.
8	**C**	We assume $P(k)$ and prove $P(k+1)$.
9	**C**	2 is prime and even.
10	**A**	That is the definition.