Test - Integration and the fundamental theorem of calculus

Year: University Year 1 · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Integration techniques

a. Find $\int x \cos x \, dx$. (2 marks)

b. Evaluate $\int_0^1 \frac{x}{1+x^2} dx$. (2 marks)

Question 2 (4 marks)

Question 2 – Partial fractions

Evaluate $\int_2^3 \frac{dx}{(x-1)(x+1)}$, giving the answer as a single logarithm.

Question 3 (4 marks)

Question 3 – Problem: area and volume

The region R is bounded by $y = x^2$, the x-axis and the line $x = 2$.

a. Find the area of R. (1 mark)
b. Find the volume when R is rotated through 2π about the x-axis. (2 marks)
c. Find the mean value of $y = x^2$ on $[0, 2]$. (1 mark)

Question 4 (4 marks)

Question 4 – Improper integrals

a. Determine whether $\int_1^\infty \frac{dx}{x^{3/2}}$ converges and find its value. (2 marks)

b. Show that $\int_1^{\infty} \frac{dx}{\sqrt{x}}$ diverges. (2 marks)

Question 5 (4 marks)

Question 5 – The fundamental theorem

a. Find $\frac{d}{dx}\int_1^{x^3} \ln t \, dt$. (2 marks)

b. Estimate $\int_0^1 x^2 dx$ using the right-endpoint Riemann sum with 4 strips. (2 marks)