Test - Functions, limits and continuity

Year: University Year 1 · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Evaluating limits

a. Evaluate $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$. (2 marks)

b. Evaluate $\lim_{x \to 0} \dfrac{\sin 2x}{\sin 3x}$. (2 marks)

Question 2 (4 marks)

Question 2 – Epsilon-delta proof

Prove from the definition that $\lim_{x \to 4}(2x + 1) = 9$.

Question 3 (4 marks)

Question 3 – Continuity

Let *f(x)=\begin{cases}x^2+a&x*.

a. Find the value of a for which f is continuous at $x = 1$. (3 marks)
b. Is f then differentiable at 1? (1 mark)

Question 4 (4 marks)

Question 4 – Problem: the shelf

A shelf of length x metres must satisfy $f(x) = x^3 + 2x - 6 = 0$ for a design.

a. Show that there is a root between 1 and 2. (2 marks)
b. Use one bisection step to find a smaller interval. (2 marks)

Question 5 (4 marks)

Question 5 – Squeeze theorem

Show that $\lim_{x \to 0} x^2 \cos \frac{1}{x} = 0$, and explain why $\lim_{x \to 0} \cos \frac{1}{x}$ does not exist.