Test - Eigenvalues and eigenvectors

Year: University Year 1 · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Eigenvalues and eigenvectors

Find the eigenvalues and corresponding eigenvectors of $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.

Question 2 (4 marks)

Question 2 – Diagonalisation and powers

Let $A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$. Find P and D with $A = PDP^{-1}$ and hence A^n.

Question 3 (4 marks)

Question 3 – Complex and repeated eigenvalues

a. Find the eigenvalues of $\begin{pmatrix} 0 & -4 \\ 1 & 0 \end{pmatrix}$. (2 marks)

b. Show that $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ has only one independent eigenvector. (2 marks)

Question 4 (4 marks)

Question 4 – Problem: customers

Each year 20% of cinema-goers switch to streaming and 10% of streaming users go back to the cinema. The transition matrix on (cinema, streaming) is $\begin{pmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{pmatrix}$.

a. Find the eigenvalues. (2 marks)
b. Find the long-term proportion using cinema. (2 marks)

Question 5 (4 marks)

Question 5 – Cayley-Hamilton

For $A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$, show that $A^2 = A + 6I$ and use this to find A^{-1}.