Multiple choice questions - Eigenvalues and eigenvectors

Year: University Year 1 · 10

Question 1 $\mathbf{v} \neq 0$ is an eigenvector if

- ◯ **A.** $A\mathbf{v} = \mathbf{0}$ only
- ◯ **B.** $A\mathbf{v} = \lambda\mathbf{v}$
- ◯ **C.** $A\mathbf{v} = \mathbf{v}^T$
- ◯ **D.** $\det A = \lambda$

Question 2 The eigenvalues of $\begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix}$ are

- ◯ **A.** 3, 5
- ◯ **B.** 8
- ◯ **C.** 15
- ◯ **D.** 0, 8

Question 3 For a 2×2 matrix the eigenvalues sum to

- ◯ **A.** the determinant
- ◯ **B.** the trace
- ◯ **C.** zero
- ◯ **D.** one

Question 4 If A has eigenvalue 3, then A^2 has eigenvalue

- ◯ **A.** 3
- ◯ **B.** 6
- ◯ **C.** 9
- ◯ **D.** 27

Question 5 A matrix is invertible iff

- ◯ **A.** 0 is an eigenvalue
- ◯ **B.** 0 is not an eigenvalue
- ◯ **C.** it is symmetric
- ◯ **D.** it is diagonal

Question 6 n distinct eigenvalues imply

- ◯ **A.** not diagonalisable
- ◯ **B.** diagonalisable

- ◯ **C.** symmetric
- ◯ **D.** singular

Question 7 Eigenvalues of $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

- ◯ **A.** 1, -1
- ◯ **B.** i, -i
- ◯ **C.** 0, 0
- ◯ **D.** 1, 1

Question 8 If $A = PDP^{-1}$ then $A^3 =$

- ◯ **A.** $P^3D^3P^{-3}$
- ◯ **B.** PD^3P^{-1}
- ◯ **C.** P^3DP^{-3}
- ◯ **D.** D^3

Question 9 The eigenvalues of an upper triangular matrix are

- ◯ **A.** its row sums
- ◯ **B.** its diagonal entries
- ◯ **C.** always 0
- ◯ **D.** unrelated to entries

Question 10 Cayley-Hamilton: a matrix

- ◯ **A.** is always diagonalisable
- ◯ **B.** satisfies its characteristic polynomial
- ◯ **C.** has real eigenvalues
- ◯ **D.** has determinant 1

Answer grid - Eigenvalues and eigenvectors

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Question	Answer	Explanation
1	**B**	Definition.
2	**A**	Diagonal entries.
3	**B**	Trace.
4	**C**	λ^2.
5	**B**	$\det A = \prod \lambda_i \neq 0$.
6	**B**	n independent eigenvectors.
7	**B**	$\lambda^2 + 1 = 0$.
8	**B**	The inner $P^{-1}P$ cancel.
9	**B**	$\det(A - \lambda I)$ is the product of diagonal differences.
10	**B**	$p(A) = 0$.