Test - Differentiation and its applications

Year: University Year 1 · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Differentiation rules

a. Differentiate $f(x) = \dfrac{x}{e^x + 1}$. (2 marks)

b. Find $\dfrac{dy}{dx}$ for $y = (\sin x)^x$ with 0. (2 marks)

Question 2 (4 marks)

Question 2 – Optimisation problem: fencing

A farmer has 120 m of fence and uses a straight river as one side of a rectangular pen. The sides at right angles to the river have length x m.

a. Show that the area is $A = x(120 - 2x)$. (1 mark)
b. Find the value of x that gives the greatest area, and prove it is a maximum. (2 marks)
c. Find the greatest area. (1 mark)

Question 3 (4 marks)

Question 3 – Mean value theorem

Let $f(x) = x^3$ on $[0, 2]$.

a. State why the mean value theorem applies. (1 mark)
b. Find the value of c in $(0, 2)$ guaranteed by the theorem. (3 marks)

Question 4 (4 marks)

Question 4 – Limits

a. Evaluate $\lim_{x \to 0} \frac{\sin x - x}{x^3}$. (3 marks)

b. Evaluate $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$ using the Maclaurin series. (1 mark)

Question 5 (4 marks)

Question 5 – Taylor approximation

(a) Find the Taylor polynomial of degree 2 of $f(x) = \sqrt{x}$ about $x = 4$. (3 marks) (b) Use it to estimate $\sqrt{4.4}$, and compare with the calculator value 2.09762. (1 mark)