Test - Differential equations

Year: University Year 1 · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – A separable equation

Solve $\dfrac{dy}{dx} = \dfrac{x}{y}$ with $y(0) = 2$, giving y explicitly for $y > 0$.

Question 2 (4 marks)

Question 2 – A first-order linear equation

Solve $y' + y = 2e^{x}$ with $y(0) = 2$.

Question 3 (4 marks)

Question 3 – A second-order equation

Solve $y'' - 4y' + 4y = 0$ with $y(0) = 1$ and $y'(0) = 3$.

Question 4 (4 marks)

Question 4 – Problem: cooling metal

A metal bar at 200°C is placed in a workshop at 20°C. Its temperature satisfies $\dfrac{dT}{dt} = -k(T - 20)$. After 10 minutes the bar is at 110°C.

a. Show that $T = 20 + 180e^{-kt}$ and find k. (3 marks)
b. Find when the bar reaches 50°C. (1 mark)

Question 5 (4 marks)

Question 5 – Forced oscillation

Find the general solution of $y'' + y' - 2y = 6e^{2x}$.