Test - Roots of polynomials, series and Maclaurin

Year: Further Maths · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Roots of a cubic

The roots of $2x^3 - 3x^2 + 4x - 5 = 0$ are α, β and γ.

a. Write down $\sum \alpha$, $\sum \alpha\beta$ and $\alpha\beta\gamma$. [2]
b. Find $\sum \alpha^2$ and explain what it tells you about the roots. [2]

Question 2 (4 marks)

Question 2 – Transformed roots

The roots of $x^3 - 2x^2 + 3x - 4 = 0$ are α, β, γ. Find a cubic equation with integer coefficients whose roots are $3\alpha - 1$, $3\beta - 1$ and $3\gamma - 1$. [4]

Question 3 (4 marks)

Question 3 – Standard sums

a. Show that $\sum_{r=1}^{n} (r^3 - r) = \frac{1}{4}(n-1)n(n+1)(n+2)$. [3]
b. Evaluate $\sum_{r=1}^{10} (r-1)r(r+1)$. [1]

Question 4 (4 marks)

Question 4 – A Maclaurin series

a. Find the Maclaurin series of $\ln(1 + 2x)$ up to the term in x^3, and state the values of x for which it is valid. [3]
b. Use your series with $x = 0.1$ to estimate $\ln 1.2$. [1]

Question 5 (4 marks)

Question 5 – Problem: a fading appeal

A charity appeal raises $\dfrac{60}{r(r+1)}$ pounds (in thousands) in week r.

a. Use the method of differences to show that the total raised in the first n weeks is $\dfrac{60n}{n+1}$ thousand pounds. [2]

b. Explain why the total never reaches £60 000, and find the first week after which the total exceeds £55 000. [2]