Test - Proof by induction

Year: Further Maths · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – A series

Prove by induction that $\sum_{r=1}^{n} r(r+2) = \frac{1}{6}n(n+1)(2n+7)$ for all positive integers n.

Question 2 (4 marks)

Question 2 – Divisibility

Prove by induction that $6^n + 4$ is divisible by 5 for all positive integers n.

Question 3 (4 marks)

Question 3 – A matrix power

Prove by induction that $\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$ for all $n \in \mathbb{Z}^+$.

Question 4 (4 marks)

Question 4 – A recurrence

A sequence has $u_1 = 5$ and $u_{n+1} = 4u_n - 3$.

a. Find u_2 and u_3. [1]
b. Prove by induction that $u_n = 4^n + 1$. [3]

Question 5 (4 marks)

Question 5 – Problem: followers on social media

After n weeks, account A has 2^n followers and account B has n^2 followers.

a. Show that the accounts have equal numbers of followers when $n = 4$, and that A

has more when $n = 5$. [1]

b. Prove by induction that $2^n > n^2$ for all $n \geq 5$. You may use the fact that $2k^2 > (k+1)^2$ for $k \geq 5$. [3]