Multiple choice questions - Proof by induction

Year: Further Maths · 10

Question 1 In an induction proof the basis step shows that:

- ◯ **A.** $P(k)$ is true for every k
- ◯ **B.** $P(n_0)$ is true for the first value n_0
- ◯ **C.** $P(k) \Rightarrow P(k+1)$
- ◯ **D.** $P(k+1)$ is false

Question 2 Which is the correct assumption step?

- ◯ **A.** Assume true for all n
- ◯ **B.** Assume true for $n = k+1$
- ◯ **C.** Assume true for some integer $k \geq 1$
- ◯ **D.** Assume $n = 1$ only

Question 3 $\sum_{r=1}^{k+1} r^3$ equals:

- ◯ **A.** $\sum_{r=1}^{k} r^3 + (k+1)^3$
- ◯ **B.** $\sum_{r=1}^{k} r^3 + k^3$
- ◯ **C.** $\left(\sum_{r=1}^{k} r^3\right)(k+1)^3$
- ◯ **D.** $\sum_{r=1}^{k} r^3 + (k+1)$

Question 4 To prove $f(n) = 7^n - 1$ divisible by 6, a useful expression is:

- ◯ **A.** $f(k+1) + f(k)$
- ◯ **B.** $f(k+1) - 7f(k)$
- ◯ **C.** $f(k+1) \div f(k)$
- ◯ **D.** $f(k+1) - k$

Question 5 If $\mathbf{M}^k = \begin{pmatrix} 1 & 3k \\ 0 & 1 \end{pmatrix}$, then $\mathbf{M}^{k+1}$ with $\mathbf{M} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ is:

- ◯ **A.** $\begin{pmatrix} 1 & 3k+3 \\ 0 & 1 \end{pmatrix}$
- ◯ **B.** $\begin{pmatrix} 1 & 3k \\ 0 & 1 \end{pmatrix}$
- ◯ **C.** $\begin{pmatrix} 1 & 9k \\ 0 & 1 \end{pmatrix}$

- ○ **D.** $\begin{pmatrix} 2 & 3k+3 \\ 0 & 2 \end{pmatrix}$

Question 6 Which statement is true for all positive integers n?

- ○ **A.** n^2+n+41 is prime
- ○ **B.** $2^n > n^2$
- ○ **C.** n^2+n is even
- ○ **D.** n^2

Question 7 For the second-order recurrence $u_{n+2} = au_{n+1} + bu_n$ a proof by induction needs:

- ○ **A.** one base case
- ○ **B.** two base cases and a two-step assumption
- ○ **C.** no base case
- ○ **D.** three base cases

Question 8 In proving $n! > 2^n$ for $n \geq 4$, the basis case is:

- ○ **A.** $n = 1$
- ○ **B.** $n = 0$
- ○ **C.** $n = 4$
- ○ **D.** $n = 5$

Question 9 $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$. When $n = 4$ the sum is:

- ○ **A.** 40
- ○ **B.** 20
- ○ **C.** 60
- ○ **D.** 30

Question 10 Which is the correct conclusion?

- ○ **A.** True for $n = k$, so true for all n
- ○ **B.** True for $n = 1$; true for $n = k \Rightarrow$ true for $n = k+1$; so true for all positive integers n
- ○ **C.** True for $n = k+1$, so done
- ○ **D.** The basis is true, so done

Answer grid - Proof by induction

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Question	Answer	Explanation
1	**B**	The basis checks the statement for the smallest value, usually $n = 1$.
2	**C**	You assume the statement for one (arbitrary) integer k, then deduce it for $k + 1$.
3	**A**	Add the next term, the $(k + 1)$th, which is $(k + 1)^3$.
4	**B**	$f(k + 1) - 7f(k) = 7^{k+1} - 1 - 7^{k+1} + 7 = 6$.
5	**A**	Multiply: top-right entry $3 + 3k = 3(k + 1)$.
6	**C**	$n(n + 1)$ is a product of consecutive integers so is even. $2^n > n^2$ fails at $n = 3$.
7	**B**	u_{k+2} depends on both u_{k+1} and u_k.
8	**C**	The statement is claimed from $n = 4$: $24 > 16$.
9	**A**	$2 + 6 + 12 + 20 = 40 = \frac{1}{3} \cdot 4 \cdot 5 \cdot 6$.
10	**B**	Both the basis and the implication must be stated.