Test - Polar coordinates

Year: Further Maths · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Polar and Cartesian

a. Convert the point $(-2\sqrt{3},\ 2)$ to polar coordinates with $-\pi$. [2]
b. Find a Cartesian equation of the curve $r = 6\sin\theta$ and describe the curve fully. [2]

Question 2 (4 marks)

Question 2 – A limaçon

The curve C has equation $r = 3 + 2\sin\theta$, $0 \leq \theta$.

a. State the greatest and least values of r, and the values of θ at which they occur. [2]
b. Find the exact area enclosed by C. [2]

Question 3 (4 marks)

Question 3 – One petal

The curve $r = 4\sin 2\theta$, $0 \leq \theta \leq \frac{\pi}{2}$, forms one petal of a rose.

a. Find the polar coordinates of the tip of the petal. [1]
b. Show that the area of the petal is 2π. [3]

Question 4 (4 marks)

Question 4 – Tangents parallel to the initial line

The cardioid C has equation $r = 4(1 + \cos\theta)$, $-\pi$.

a. Find the values of θ, other than π, where the tangent to C is parallel to the initial line. [3]
b. Give the Cartesian coordinates of the point with $\theta > 0$. [1]

Question 5 (4 marks)

Question 5 – Problem: a lawn around a pond

A circular lawn has boundary $r = 4$ (in metres) about a sprinkler at the pole. A pond has the cardioid shape $r = 4(1 - \cos\theta)$. The region G inside the circle and outside the pond is grass.

a. Find the values of θ at which the boundaries meet. [1]
b. Show that the area of grass in the region $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ is $32 - 4\pi$ m². [3]