Test - Matrices and linear transformations

Year: Further Maths · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	4	4	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Products and determinants

$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ -2 & 5 \end{pmatrix}$.

a. Find $\mathbf{AB}$. [2]
b. Find $\det \mathbf{A}$, $\det \mathbf{B}$ and $\det(\mathbf{AB})$, and verify that $\det(\mathbf{AB}) = \det \mathbf{A} \det \mathbf{B}$. [2]

Question 2 (4 marks)

Question 2 – A singular matrix

, where k is a constant.

a. Show that $\det \mathbf{N} = k^2 - k - 2$. [2]
b. Find the values of k for which $\mathbf{N}$ is singular. [1]
c. When $k = 3$, find the volume of the image of a cube of side 2 under $\mathbf{N}$. [1]

Question 3 (4 marks)

Question 3 – Composite transformation

T_1 is a rotation of 90° clockwise about O and T_2 is a reflection in the y-axis.

a. Write down the matrices of T_1 and T_2. [2]
b. Find the matrix of T_1 followed by T_2 and describe this single transformation. [2]

Question 4 (4 marks)

Question 4 – Invariant lines

The matrix $\mathbf{M} = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$ represents a transformation T.

a. Show that every point on the line $y = -x$ is invariant under T. [1]
b. Find the other invariant line of T that passes through the origin. [2]
c. Explain whether this second line is a line of invariant points. [1]

Question 5 (4 marks)

Question 5 – Problem: a map of a field

On a map, a triangular field has vertices $O(0, 0)$, $P(4, 0)$ and $Q(0, 2)$, in units of 100 m. A software error transforms the map with the matrix $\mathbf{M} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$.

a. Find the images of P and Q. [1]
b. Find the area of the field and the area of its image on the transformed map. [2]
c. A tree appears at $(5, 5)$ on the transformed map. Find its true position. [1]