Test - Hyperbolic functions

Year: Further Maths · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	4	4	5	3	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Identities and a quadratic

(a) Prove, using exponentials, that $\cosh 2x = 2\cosh^2 x - 1$. [2]

(b) Solve $\cosh 2x = 3\cosh x - 2$. [2]

Question 2 (4 marks)

Question 2 – An exponential equation

Solve $2\sinh x + \cosh x = 2$, giving the answer as a natural logarithm. [4]

Question 3 (4 marks)

Question 3 – Differentiation

Differentiate with respect to x:

a. $y = x\sinh 2x$ [2]
b. $y = \cosh^2 3x$, giving the answer in the form $k\sinh 6x$ [2]

Question 4 (5 marks)

Question 4 – Integration

(a) Find the exact value of $\displaystyle\int_0^2 \frac{dx}{\sqrt{x^2+4}}$. [2]

(b) Use integration by parts to show that $\displaystyle\int_0^1 x\sinh x\,dx = e^{-1}$. [3]

Question 5 (3 marks)

Question 5 – Problem: the hanging cable

A cable hangs in the shape $y = 10\cosh\dfrac{x}{10}$ between two posts at $x = \pm 15$ (metres).

a. Find the height of each post. [1]

b. Find the length of the cable, using the fact that its length is $\int \cosh\dfrac{x}{10}\,dx$ over the span. [2]