Test answers - Further vectors

Year: Further Maths · Duration: 60 minutes · /20

Answer to Question 1 (5 marks)

Method: the scalar product is $\mathbf{a}\cdot\mathbf{b}=a_1b_1+a_2b_2+a_3b_3=|\mathbf{a}||\mathbf{b}|\cos\theta$. Two non-zero vectors are perpendicular exactly when their scalar product is 0. the vector product $\mathbf{a}\times\mathbf{b}$ is perpendicular to both vectors and has magnitude $|\mathbf{a}||\mathbf{b}|\sin\theta$, the area of the parallelogram with sides $\mathbf{a}$ and $\mathbf{b}$. Compute it as a 3×3 determinant. Mark scheme as shown in brackets.

a. $\mathbf{a}\cdot\mathbf{b}=3+0-2=1$ [1]; $\cos\theta=\dfrac{1}{\sqrt{6}\sqrt{13}}$ [1]; $\theta=$ **83.5°** [1]

b. $\mathbf{a}\times\mathbf{b}=(4,-5,-6)$ [1]; area $=\frac{1}{2}\sqrt{16+25+36}=\frac{1}{2}\sqrt{77}=$ **4.39** [1]

Answer to Question 2 (5 marks)

Method: substitute the general point of the line into the equation of the plane to find the parameter. the angle between a line with direction $\mathbf{d}$ and a plane with normal $\mathbf{n}$ satisfies $\sin\varphi=\dfrac{|\mathbf{d}\cdot\mathbf{n}|}{|\mathbf{d}||\mathbf{n}|}$ (the complement of the angle with the normal). The angle between two planes is the angle between their normals. Mark scheme as shown in brackets.

a. $3(2+\lambda)+(-1+2\lambda)-(-2\lambda)=7$ [1] gives $5+7\lambda=7$, $\lambda=\frac{2}{7}$ [1]. Point $\left(\frac{16}{7},-\frac{3}{7},-\frac{4}{7}\right)$ [1]

b. $\sin\varphi=\dfrac{|3+2+2|}{3\sqrt{11}}$ [1] $=0.7035$, $\varphi=$ **44.7°** [1]

Check: $3\times\frac{16}{7}-\frac{3}{7}+\frac{4}{7}=\frac{49}{7}=7$, so the point lies in the plane.

Answer to Question 3 (4 marks)

Method: a plane with normal $\mathbf{n}$ through the point A has equation $\mathbf{r}\cdot\mathbf{n}=\mathbf{a}\cdot\mathbf{n}$, that is $n_1x+n_2y+n_3z=d$. A normal to a plane containing two directions is their vector product. Mark scheme as shown in brackets.

a.  [1]; cross product $=(-3,-3,0)$ [1]; plane $x+y=2$ (using A) [1]

b. $\dfrac{|3+3-2|}{\sqrt{2}}=\dfrac{4}{\sqrt{2}}=2\sqrt{2}$ [1]

Check: B: $2+0=2$ and C: $0+2=2$. The plane is vertical (it contains the z direction).

Answer to Question 4 (3 marks)

Method: the distance from P to the line through A with direction $\mathbf{d}$ is , the height of the parallelogram with sides and $\mathbf{d}$. Mark scheme as shown in brackets.

a. [1]; [1]; distance $= \dfrac{\sqrt{20}}{\sqrt{6}} =$ **1.83** [1]

Answer: the shortest distance is $\sqrt{\frac{20}{6}} = \sqrt{\frac{10}{3}} \approx 1.83$.

Check with the foot of the perpendicular: $\lambda = \dfrac{(1,3,2)\cdot(1,1,2)}{6} = \dfrac{4}{3}$, giving the foot $\left(\frac{4}{3}, \frac{7}{3}, \frac{8}{3}\right)$ and the same distance $\sqrt{\frac{10}{3}}$.

Answer to Question 5 (3 marks)

Method: the angle between a line with direction $\mathbf{d}$ and a plane with normal $\mathbf{n}$ satisfies $\sin\varphi = \dfrac{|\mathbf{d}\cdot\mathbf{n}|}{|\mathbf{d}||\mathbf{n}|}$ (the complement of the angle with the normal). The angle between two planes is the angle between their normals. Mark scheme as shown in brackets.

a. Normals $(3,0,4)$ and $(0,0,1)$: $\cos\theta = \dfrac{4}{5}$, $\theta =$ **36.9**° [1]

b. $\sin\varphi = \dfrac{|-3-8|}{\sqrt{5}\times 5} = \dfrac{11}{5\sqrt{5}} = 0.984$ [1], $\varphi =$ **79.7**° [1]

So the sunlight strikes the panel at almost $80°$, close to the ideal $90°$.