Test - Further calculus

Year: Further Maths · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	5	5	3	3	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Improper integrals

(a) Evaluate $\int_0^{\infty} 6e^{-2x}\,dx$. [2]

(b) Show that $\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx$ diverges. [2]

Question 2 (5 marks)

Question 2 – Inverse trigonometric integrals

(a) Show that $\int_0^{1} \frac{dx}{x^2+2x+2} = \arctan 2 - \frac{\pi}{4}$. [3]

(b) Evaluate $\int_0^{1} \frac{dx}{\sqrt{4-x^2}}$ exactly. [2]

Question 3 (5 marks)

Question 3 – Volume of revolution

The region R is bounded by $y = \frac{1}{\sqrt{1+x^2}}$, the x-axis, the y-axis and the line $x = 1$.

a. Show that the volume formed when R is rotated about the x-axis is $\frac{\pi^2}{4}$. [3]

b. Find the area of the cross-section of the solid at $x = 1$ and the volume of a cylinder of that radius and length 1. [2]

Question 4 (3 marks)

Question 4 – Mean value

Find the mean value of $f(x) = 4x - x^2$ over the interval $[0, 4]$. [3]

Question 5 (3 marks)

Question 5 – Problem: the planter

A planter is made by rotating the curve $y = \frac{x^2}{4}$ about the y-axis, where x and y are in centimetres, up to a depth of 9 cm.

a. Find the radius of the top of the planter. [1]
b. Find the volume of compost it holds, to the nearest cm^3. [2]