Test - Differential equations

Year: Further Maths · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	6	4	3	3	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – First order equation

Solve $\frac{dy}{dx} - \frac{3y}{x} = x^2$ for $x > 0$, given that $y = 2$ when $x = 1$. [4]

Question 2 (6 marks)

Question 2 – Second order with initial conditions

(a) Find the general solution of $y'' - 5y' + 6y = 4e^x$. [4]

(b) Given that $y = 5$ and $y' = 7$ when $x = 0$, find the particular solution. [2]

Question 3 (4 marks)

Question 3 – Complex roots

Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 20y = 0$, given that $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 0$. [4]

Question 4 (3 marks)

Question 4 – Simple harmonic motion

A particle moves with $\ddot{x} = -25x$ and amplitude 0.2 m.

a. Find the period. [1]
b. Find the greatest speed. [1]
c. Find the speed when $x = 0.12$ m. [1]

Question 5 (3 marks)

Question 5 – Problem: the cooling engine

A hot engine at 75°C cools in a garage at 15°C so that $\frac{dT}{dt} = -0.05(T - 15)$, where t is in minutes.

a. Show that $T = 15 + 60e^{-0.05t}$. [2]
b. Find how long the engine takes to cool to 30°C. [1]