Test - De Moivre's theorem and roots of unity

Year: Further Maths · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	4	5	5	3	3	**20**
Mark						

Show your working.

Question 1 (4 marks)

Question 1 – Powers of complex numbers

(a) Express $1 - i$ in modulus-argument form. [2]

(b) Use de Moivre's theorem to show that $(1 - i)^{10} = -32i$. [2]

Question 2 (5 marks)

Question 2 – Powers of sine

(a) By expanding $\left(z - \frac{1}{z}\right)^4$ where $z = \cos\theta + i\sin\theta$, show that $\sin^4\theta = \frac{1}{8}(\cos 4\theta - 4\cos 2\theta + 3)$. [3]

(b) Hence find the exact value of $\int_0^{\pi/4} \sin^4\theta \, d\theta$. [2]

Question 3 (5 marks)

Question 3 – Roots of a complex number

(a) Solve $z^3 = -27i$, giving the roots in the form $r(\cos\theta + i\sin\theta)$ with $-\pi$. [3]

(b) Show the roots on an Argand diagram and find the area of the triangle they form. [2]

Question 4 (3 marks)

Question 4 – Solving a cosine equation

Using $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, solve $\cos 3\theta = \cos\theta$ for $0 \le \theta \le \pi$. [3]

Question 5 (3 marks)

Question 5 – Problem: the hexagonal tiles

A tile is a regular hexagon whose corners are the sixth roots of unity, scaled so that the distance from the centre to each corner is 5 cm.

a. Write the corners as complex numbers in exponential form. [1]
b. Find the length of one side, and the area of the tile to 3 significant figures. [2]