Test - Complex numbers

Year: Further Maths · Duration: 60 minutes · /20

Name:	Date:	**Mark: /20**

Question	1	2	3	4	5	Total
marks	3	4	4	6	3	**20**
Mark						

Show your working.

Question 1 (3 marks)

Question 1 – Cartesian arithmetic

Given $z_1 = 2 + 3i$ and $z_2 = 1 - i$:

a. Find $\dfrac{z_1}{z_2}$ in the form $a + bi$. [2]
b. Find $|z_1 z_2|$. [1]

Question 2 (4 marks)

Question 2 – Roots of a cubic

$f(z) = z^3 - 3z^2 + 7z - 5$.

a. Show that $z = 1$ is a root of $f(z) = 0$. [1]
b. Solve $f(z) = 0$ completely. [2]
c. Sketch the roots on an Argand diagram. [1]

Question 3 (4 marks)

Question 3 – Modulus and argument

$u = -3 + 3\sqrt{3}\,i$ and $v = 2 - 2i$.

a. Find the modulus and argument of u and of v. [2]
b. Find $\left|\dfrac{u}{v}\right|$ and $\arg\dfrac{u}{v}$, as exact values. [2]

Question 4 (6 marks)

Question 4 – Problem: the sensors

Two sensors are at the points $A = 2$ and $B = 6i$ on an Argand diagram (distances in km). A boat is the same distance from both sensors and is also exactly 5 km from A.

a. Write the first condition as an equation in z and show that its Cartesian equation is $x - 3y = -8$. [3]
b. Find the exact coordinates of the two positions where the boat could be. [3]

Question 5 (3 marks)

Question 5 – Quadratic with a complex root

The quadratic equation $z^2 + pz + q = 0$ has real coefficients and one root is $2 - i$.

a. Write down the other root. [1]
b. Find p and q. [2]